GCE

Mathematics

2300QS/1300QS

Summer 2023 examinations

AS	Unit 1	Pure Mathematics A	Wednesday, 17 May 2023
AS	Unit 2	Applied Mathematics A	Thursday, 25 May 2023
A2	Unit 3	Pure Mathematics B	Thursday, 8 June 2023
A2	Unit 4	Applied Mathematics B	Tuesday, 13 June 2023

Advance Information

General information for students and teachers

This advance information provides the focus of the content of the Summer 2023 examination papers.

It does not apply to any other examination series.

It is intended to support revision.

It may be used at any time from the date of release.

It must not be taken into the examination.

Released: 06 February 2023

Subject information for students and teachers

It is important that this advance information is read with reference to the detailed subject content in the specification, which is available at: WJEC GCE Mathematics specification

This advance information covers Unit 1 (2300U10-1), Unit 2 (2300U20-1), Unit 3 (1300U30-1) and Unit 4 (1300U40-1).

The format and structure of the examination papers remains unchanged.

The following areas of content are suggested as key areas of focus for revision and final preparation, in relation to the Summer 2023 examinations. Please note, while advance information is intended to help guide and prioritise revision, to support exam performance and progression, revision plans should still take account of everything that has been taught, as content not listed in the advance information may still be assessed.

The information for each unit is presented in the order it appears in the specification, not in question order in the examination.

The aim should still be to cover all specification content in teaching and learning.

In addition to advance information for Unit 1 to Unit 4, all GCE Mathematics examination papers in 2023 will include a list of additional formulae. This will be given on page 2 of each examination paper. The list of additional formulae forms part of the advance information for GCE Mathematics and is presented at the end of this document, following the advance information for Unit 4.

AS Unit 1 Pure Mathematics A

Topics
2.1.1 Proof
Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including disproof by counter example.
2.1.2 Algebra and Functions
Understand and use the laws of indices for all rational exponents. Use and manipulate surds, including rationalising the denominator.
Work with quadratic functions and their graphs. The discriminant of a quadratic function, including the conditions for real roots and repeated roots. Completing the square. Solution of quadratic equations in a function of the unknown.
Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation.
Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions. Express solutions through the correct use of 'and' and 'or', or through set notation.
Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the Factor Theorem.
Understand and use graphs of functions; sketch curves defined by simple equations, including polynomials. $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$, including their vertical and horizontal asymptotes. Interpret algebraic solutions of equations graphically. Use intersection points of graphs of curves to solve equations.
Understand the effect of simple transformations on the graph of $y = f(x)$ including sketching associated graphs: $y = f(x + a)$
2.1.3 Coordinate geometry in the (x, y) plane
Understand and use the equation of a straight line, including the forms $y = mx + c$, $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular.
Understand and use the coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle. Use of the following circle properties: the angle in a semicircle is a right angle.

Topics
2.1.4 Sequences and Series - The Binomial Theorem
Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n . The notations $n!$, $\binom{n}{r}$ and $n\mathrm{C}r$.
2.1.5 Trigonometry
Understand and use the definitions of sine, cosine and tangent for all arguments.
Understand and use the sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab\sin C$.
Understand and use the sine, cosine and tangent functions. Understand and use their graphs, symmetries and periodicity.
Understand and use $\cos^2\theta + \sin^2\theta = 1$.
Solve simple trigonometric equations in a given interval, including quadratic equations in $\sin$, $\cos$ and $\tan$.
2.1.6 Exponentials and logarithms
Know and use the function a^x and its graph, where a is positive. Know and use the function e^x.
Know that the gradient of e^{kx} is equal to $k\mathrm{e}^{kx}$ and hence understand why the exponential model is suitable in many applications.
Know and use the definition of $\log_a x$ as the inverse of a^x , where a is positive and $x \geq 0$. Know and use the function $\ln x$. Know and use $\ln x$ as the inverse function of e^x.
Understand and use the laws of logarithms. $\log_a x + \log_a y = \log_a (xy)$ $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ $k \log_a x = \log_a (x^k)$ (including, for example $k = -1$, $k = -\frac{1}{2}$)
Solve equations in the form $a^x = b$.
Understand and use exponential growth and decay; use in modelling (examples may include drug concentration decay).

Topics
2.1.7 Differentiation
Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y). Differentiation from first principles for small positive integer powers of x.
Differentiate x^n for rational n, and related constant multiples, sums and differences.
Apply differentiation to find gradients and tangents. Identify where functions are increasing or decreasing.
2.1.8 Integration
Know and use the Fundamental Theorem of Calculus.
Integrate x^n (excluding $n = -1$) and related sums, differences and constant multiples.
Evaluate definite integrals. Use a definite integral to find the area under a curve.
2.1.9 Vectors
Use vectors in two dimensions.
Perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.
Understand and use position vectors; calculate the distance between points represented by position vectors. Use vectors to solve problems in pure mathematics.

AS Unit 2 Applied Mathematics A

Please note: Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the **FULL** specification content of Unit 1.

Topics
STATISTICS
2.2.1 Statistical Sampling
Understand and use the terms 'population' and 'sample'. Use samples to make informal inferences about the population. Understand and use sampling techniques, including simple random sampling.
Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population.
2.2.2 Data presentation and interpretation
Interpret measures of central tendency and variation, extending to standard deviation. Be able to calculate standard deviation.
2.2.3 Probability
Understand and use mutually exclusive and independent events when calculating probabilities.
Use Venn diagrams to calculate probabilities.
2.2.4 Statistical distributions
Understand and use simple, discrete probability distributions. Understand and use, • the binomial distribution, as a model • the Poisson distribution, as a model • the discrete uniform distribution, as a model (Calculation of mean and variance of discrete random variables is excluded.)
Calculate probabilities using • the binomial distribution, • the Poisson distribution.
Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial, Poisson or discrete uniform model may not be appropriate.

Topics
STATISTICS
2.2.5 Statistical hypothesis testing
Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, *p*-value.
Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context. Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis.

Topics
MECHANICS
2.2.6 Quantities and units in mechanics
Understand and use fundamental quantities and units in the S.I. system; length, time and mass.
Understand and use derived quantities and units: velocity, acceleration, force, weight.
2.2.7 Kinematics
Understand and use the language of kinematics: position, displacement, distance travelled, velocity, speed, acceleration.
Understand, use and interpret graphs in kinematics for motion in a straight line: velocity against time and interpretation of the gradient and the area under the graph.
Understand and use the formulae for constant acceleration for motion in a straight line.
Use calculus in kinematics for motion in a straight line.
2.2.8 Forces and Newton's laws
Understand the concept of a force. Understand and use Newton's first law.
Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors).
Understand and use weight and motion in a straight line under gravity; gravitational acceleration, g, and its value in S.I. units to varying degrees of accuracy. (The inverse square law for gravitation is not required and g may be assumed to be constant, but learners should be aware that g is not a universal constant but depends on location.)
Understand and use Newton's third law. Equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors) Applications to problems involving smooth pulleys and connected particles.
2.2.9 Vectors
Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.
Use vectors to solve problems in context, including forces.

A2 Unit 3 Pure Mathematics B

Please note: Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the **Summer 2022 adapted** specification content of Unit 1.

Topics
2.3.2 Algebra and Functions
Simplify rational expressions, including by factorising and cancelling and by algebraic division (by linear expressions only).
Sketch curves defined by the modulus of a linear function.
Understand and use composite functions; inverse functions.
Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear).
2.3.3 Coordinate geometry in the (x, y) plane
Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.
2.3.4 Sequences and Series
Understand and use the binomial expansion of $(a+bx)^n$, for any rational n, including its use for approximation. Be aware that the expansion is valid for $\left\|\frac{bx}{a}\right\|<1$ (proof not required).
Understand and work with arithmetic sequences and series, including the formulae for the nth term and the sum to n terms.
Understand and work with geometric sequences and series, including the formulae for the nth term and the sum of a finite geometric series. The sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation.
Use sequences and series in modelling.

Topics
2.3.5 Trigonometry
Know and use exact values of $\sin$ and $\cos$ for $0,\ \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2},\ \pi$ and multiples thereof.
Understand and use the definitions of sec, cosec, cot, $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. Understand the relationships of all of these to sin, cos and tan and understand their graphs, ranges and domains.
Understand and use $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$.
Use of formulae for $\cos(A \pm B)$.
Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$.
2.3.6 Differentiation
Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves, and points of inflection.
Differentiate e^{kx}, $\sin kx$, $\cos kx$, and related sums, differences and constant multiples.
Apply differentiation to find points of inflection.
Differentiate using the product rule, the quotient rule and the chain rule.
Differentiate simple functions and relations defined implicitly, for first derivative only.
Construct simple differential equations in pure mathematics.
2.3.7 Integration
Integrate $e^{kx}, \frac{1}{x}$ and related sums, differences and constant multiples.
Use a definite integral to find the area between two curves.
Carry out simple cases of integration by substitution and integration by parts. Understand these methods as the reverse processes of the chain rule and the product rule respectively. Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated. Integration by parts includes more than one application of the method but excludes reduction formulae.
Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions. (Separation of variables may require factorisation involving a common factor.)

Topics
2.3.8 Numerical Methods
Locate roots of $f(x) = 0$ by considering changes in sign of $f(x)$ in an interval of x in which $f(x)$ is sufficiently well-behaved.
Solve equations approximately using simple iterative methods. Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = g(x_n)$. Understand how such methods can fail.
Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between.
Use numerical methods to solve problems in context.

A2 Unit 4 Applied Mathematics B

Please note: Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the:

- **Summer 2022 adapted** specification content of Unit 1 and Unit 2
- **FULL** specification content of Unit 3.

Topics
STATISTICS
2.4.1 Probability
Understand and use conditional probability, including the use of tree diagrams, Venn diagrams and two-way tables.
Understand and use the conditional probability formula: $P(A \cap B) = P(A)\,P(B\|A) = P(B)\,P(A\|B)$.
Modelling with probability.
2.4.2 Statistical distributions
Understand and use the continuous uniform distribution and Normal distributions as models. Find probabilities using the Normal distribution. Link to mean and standard deviation.
2.4.3 Statistical hypothesis testing
Understand and apply statistical hypothesis testing to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given *p*-value or critical value. (The calculation of correlation coefficients is excluded.)
Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance, and interpret the results in context.

Topics
DIFFERENTIAL EQUATIONS AND MECHANICS
2.4.4 Trigonometry
Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.
2.4.5 Differentiation
Construct simple differential equations in context (contexts may include kinematics).
2.4.6 Integration
Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions.
Interpret the solution of a differential equation in the context of solving a problem; includes links to kinematics.
2.4.7 Quantities and units in mechanics
Understand and use derived quantities and units for moments.
2.4.8 Kinematics
Extend the use of calculus in kinematics for motion in a straight line to 2 dimensions using vectors.
Model motion under gravity in a vertical plane using vectors; projectiles.
2.4.9 Forces and Newton's laws
Extend Newton's second law to situations where forces need to be resolved (restricted to two dimensions).
Resolve forces in two dimensions.
Understand and use addition of forces; resultant forces; dynamics for motion in a plane.
Understand and use the $F \leq \mu R$ model for friction. The coefficient of friction. The motion of a body on a rough surface. Limiting friction and statics.
2.4.10 Moments
Understand and use moments in simple static contexts.
2.4.11 Vectors
Use vectors to solve problems in context, including kinematics.

Additional Formulae for 2023

Laws of Logarithms

$$\log_a x + \log_a y \equiv \log_a (xy)$$

$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y}\right)$$

$$k \log_a x \equiv \log_a \left(x^k\right)$$

Sequences

General term of an arithmetic progression:

$$u_n = a + (n-1)d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

Mensuration

For a circle of radius, r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Calculus and Differential Equations

Differentiation

Function	Derivative
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f\left(g(x)\right)$	$f'\left(g(x)\right)g'(x)$

Integration

Function	Integral
$f'\left(g(x)\right)g'(x)$	$f\left(g(x)\right) + c$

Area under a curve $= \int_a^b y \, \mathrm{d}x$

End of advance information