GCSE

WJEC GCSE in
MATHEMATICS
MATHEMATICS - NUMERACY

ACCREDITED BY WELSH GOVERNMENT

GUIDANCE FOR TEACHING

Teaching from 2015

This Welsh Government regulated qualification is not available to centres in England.

Teachers' Guide for GCSE Mathematics – Numeracy and GCSE Mathematics

The following is an extract from the published specification for GCSE Mathematics, giving the content for Foundation, Intermediate and Higher tiers.
The full version can found on www.wjec.co.uk
Teachers are reminded that it is the specification document, and not the Specimen Assessment Materials, which should form a basis for a scheme of learning.

Foundation tier

Candidates entered for GCSE Mathematics will be expected to be familiar with the knowledge, skills and understanding implicit in GCSE Mathematics – Numeracy.

Foundation tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding number and place value	
Reading and writing whole numbers of any magnitude expressed in figures or words. Rounding whole numbers to the nearest 10, 100, 1000, etc. Understanding place value and decimal places. Rounding decimals to the nearest whole number or a given number of decimal places.	
Using the equivalences between decimals, fractions, ratios and percentages. Converting numbers from one form into another. Ordering and comparing whole numbers, decimals, fractions and percentages. Understanding and using directed numbers, including ordering directed numbers.	
Understanding number relationships and methods of calculation	
Using the common properties of numbers, including odd, even, multiples, factors, primes. Expressing numbers as the product of their prime factors. Using the terms square, square root and cube. The use of index notation for positive integral indices. Interpreting numbers written in standard form in the context of a calculator display.	

Foundation tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Using the facilities of a calculator to plan a calculation and evaluate expressions. Using addition, subtraction, multiplication, division, square and square root. Knowing how a calculator orders its operations. (Candidates will not be expected to list the key depressions that they have made.) Using calculators effectively and efficiently. Reading a calculator display correct to a specified number of decimal places.	
Understanding and using number operations and the relationships between them, including inverse operations and the hierarchy of operations. Addition, subtraction, multiplication and division of whole numbers, decimals, fractions and negative numbers. Finding a fraction or percentage of a quantity. Expressing one number as a fraction or percentage of another. Calculating fractional and percentage changes (increase and decrease). Calculating using ratios in a variety of situations; proportional division. The use of a non-calculator method to multiply and divide whole numbers up to and including the case of multiplication and division of a three-digit number by a two-digit number.	
Recognising that recurring decimals are exact fractions, and that some exact fractions are recurring decimals.	
Estimating and approximating solutions to numerical calculations. Using estimation in multiplication and division problems with whole numbers to obtain approximate answers, e.g. by first rounding the numbers involved to 1 significant figure. Candidates must show sufficient working in order to demonstrate how they have obtained their estimate.	

Foundation tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Solving numerical problems	
Interpretation and use of mathematical information presented in written or visual form when solving problems, e.g. TV programme schedules, bus/rail timetables, distance charts, holiday booking information. Money: The basic principles of personal and household finance, including fuel and other bills, hire purchase, discount, VAT, taxation, best buys, wages and salaries, loan repayments, mortgages, budgeting, exchange rates and commissions. Simple interest. Profit and loss. Foreign currencies and exchange rates. Carrying out calculations relating to enterprise, saving and borrowing, investing, appreciation and depreciation.	See the section in this teachers' guide on vocabulary of finance.
Giving solutions in the context of a problem, interpreting the display on a calculator. Interpreting the display on a calculator. Knowing whether to round up or down as appropriate.	
Understanding and using Venn diagrams to solve problems.	Venn diagrams is a new topic in these specifications. See the section on new content. This also covers set notation, and what is needed at each tier.

Foundation tier – Algebra

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using functional relationships	
Recognition, description and continuation of patterns in number. Description, in words, of the rule for the next term of a sequence.	Finding the *n*th term of a sequence where the rule is linear. Generating linear sequences given the *n*th term rule.
Construction and interpretation of conversion graphs. Construction and interpretation of travel graphs. Construction and interpretation of graphs that describe real-life situations. Interpretation of graphical representation used in the media, recognising that some graphs may be misleading.	
Using coordinates in 4 quadrants. Drawing and interpreting the graphs of $x = a$, $y = b$, $y = ax + b$.	
Understanding and using equations and formulae	
Substitution of positive and negative whole numbers, fractions and decimals into simple formulae expressed in words or in symbols. Understanding the basic conventions of algebra. Collection of like terms. Expansion of $a(bx + c)$, where a, b and c are integers. Formation and manipulation of linear equations.	The basic conventions of algebra, collecting terms, expanding brackets and solving equations can all be assessed on GCSE Mathematics - Numeracy. Procedural questions (out of context) involving this algebra will be assessed on GCSE Mathematics.

Foundation tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using properties of shape	
The geometrical terms: point, line, plane, parallel, right angle, clockwise and anticlockwise turns, perpendicular, horizontal, vertical, acute, obtuse and reflex angles, face, edge and vertex. Vocabulary of triangles, quadrilaterals and circles: isosceles, equilateral, scalene, exterior/interior angle, diagonal, square, rectangle, parallelogram, rhombus, kite, trapezium, polygon, pentagon, hexagon, radius, diameter, tangent, circumference, chord, arc, sector, segment. Simple solid figures: cube, cuboid, cylinder, cone and sphere. Interpretation and drawing of nets. Using and drawing 2-D representations of 3-D shapes, including the use of isometric paper.	The vocabulary of shapes is included in GCSE Mathematics - Numeracy, so that the words and concepts can be used in this GCSE, as well as in the Mathematics GCSE.
Accurate use of ruler, pair of compasses and protractor. (Lengths accurate to 2mm and angles accurate to 2°.) Bisecting a given line, bisecting a given angle. Constructing 2-D shapes from given information.	Use of ruler and pair of compasses to do constructions. Construction of triangles, quadrilaterals and circles.
Candidates will be expected to bisect using a pair of compasses and a ruler in GCSE Mathematics - Numeracy, but constructing angles and shapes can only appear on GCSE Mathematics.	Simple description of symmetry in terms of reflection in a line/plane or rotation about a point. Order of rotational symmetry.
Angles at a point. Angles at a point on a straight line. Opposite angles at a vertex. Parallel lines. Corresponding, alternate and interior angles. Angle properties of triangles. Using the fact that the angle sum of a triangle is 180°.	Using the fact that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices. Using angle properties of equilateral, isosceles and right-angled triangles; understand congruence; explain why the angle sum of any quadrilateral is 360°. Regular and irregular polygons. Sum of the interior and sum of the exterior angles of a polygon.

Foundation tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using properties of position, movement and transformation	
	Finding the coordinates of points identified by geometrical information, for example, finding the coordinates of the mid-point of the line segment *AB*, given points *A* and *B*; finding the coordinates of the fourth vertex of a parallelogram, given the coordinates of the other three vertices. Location determined by distance from a given point and angle made with a given line.
Questions involving the centre of enlargement can appear at Foundation, Intermediate or Higher tiers.	Transformations, including: • Reflection • Rotation through 90°, 180°, 270°. Clockwise or anticlockwise rotations; centre of rotation • Enlargement with positive scale factors • Translation. Candidates will be expected to draw the image of a shape under transformation.
Solving problems in the context of tiling patterns and tessellation.	
Using and interpreting maps. Interpretation and construction of scale drawings. Scales may be written in the form 1 cm represents 5 m, or 1:500. Use of bearings. (Three figure bearings will be used e.g. 065°, 237°.)	

Foundation tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	**GCSE Mathematics only***
Understanding and using measures	
Standard metric units of length, mass and capacity. The standard units of time; the 12- and 24- hour clock. (The notation for the 12- and 24- hour clock will be 1:30 p.m. and 13:30.) Knowledge and use of the relationship between metric units of length, mass, capacity, area and volume. Making sensible estimates of measurements in everyday situations, recognising the appropriateness of units in different contexts. Conversion between the following metric and Imperial units: km - miles; cm, m - inches, feet; kg - lb; litres - pints, gallons. Candidates will be expected to know the following approximate equivalences: 8km $\approx$ 5 miles, 1kg $\approx$ 2·2 lb, 1 litre $\approx$ 1·75 pints	Candidates will need to know these metric to Imperial conversions. Any others will be given in the examination papers.
Reading and interpreting scales, including decimal scales.	
Using compound measures including speed. Using compound measures such as m/s, km/h, mph and mpg.	
Estimating of the area of an irregular shape drawn on a square grid. Calculating: - perimeter and area of a square, rectangle, triangle, parallelogram, trapezium, circle, semicircle and composite shapes. - surface area, cross-sectional area and volume of cubes and cuboids.	

Foundation tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using the statistical problem solving process: specifying the problem/planning; collecting, processing and representing data; interpreting and discussing results.	
Specifying the problem and planning	
Specifying and testing hypotheses, taking account of the limitations of the data available. Designing and criticising questions for a questionnaire, including notions of fairness and bias.	
Processing, representing and interpreting data	
Sorting, classification and tabulation of qualitative (categorical) data or discrete (ungrouped) data. Understanding and using tallying methods.	
Constructing and interpreting pictograms, bar charts and pie charts for qualitative data. Constructing and interpreting vertical line diagrams for discrete data. Constructing line graphs for the values of a variable at different points in time; understanding that intermediate values in a line graph may or may not have meaning. Temperature charts. Constructing and interpreting scatter diagrams for data on paired variables.	

Foundation tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Mean, median and mode for a discrete (ungrouped) frequency distribution. Comparison of two distributions using one measure of central tendency (i.e. the mean or the median). Modal category for qualitative data. Calculating or estimating the range applied to discrete data. Drawing 'by eye' a line of 'best fit' on a scatter diagram, understanding and interpreting what this represents.	
Discussing results	
Recognising that graphs may be misleading. Looking at data to find patterns and exceptions. Drawing inferences and conclusions from summary measures and data representations, relating results back to the original problem. Drawing of conclusions from scatter diagrams; using terms such as positive correlation, negative correlation, little or no correlation. Appreciating that correlation does not imply causality.	

Foundation tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Estimating and calculating the probabilities of events	
Understanding and using the vocabulary of probability, including notions of uncertainty and risk. The terms 'fair', 'evens', 'certain', 'likely', 'unlikely ' and 'impossible'.	Understanding and using the probability scale from 0 to 1. Use of: the probability of an event not occurring is one minus the probability that it occurs. (Probabilities must be expressed as fractions, decimals or percentages.)
	Estimating the probability of an event as the proportion of times it has occurred. Relative frequency. An understanding of the long-term stability of relative frequency is expected. Calculating theoretical probabilities based on equally likely outcomes. Estimating probabilities based on experimental evidence. Comparing an estimated probability from experimental results with a theoretical probability.
	Identifying all the outcomes of a combination of two experiments, *e.g. throwing two dice;* use tabulation, Venn diagrams, or other diagrammatic representations of compound events.
	Knowledge that the total probability of all the possible outcomes of an experiment is 1.

Venn diagrams and accompanying set notation is explained in more detail in the new content section of this teachers' guide. Note that any Venn diagram question that assesses probability can only appear on GCSE Mathematics.

Content in standard text:
Even though this content is included at the Intermediate tier, it is expected that candidates will be confident and competent in this content at this level. This content can be assessed implicitly at Higher and Intermediate tier but we wouldn't assess this content directly.

Intermediate tier

Foundation tier content is in standard text.
Intermediate tier content which is in addition to foundation tier content is in underlined text.

Candidates entered for GCSE Mathematics will be expected to be familiar with the knowledge, skills and understanding implicit in GCSE Mathematics – Numeracy.

Intermediate tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding number and place value	
Reading and writing whole numbers of any magnitude expressed in figures or words. Rounding whole numbers to the nearest 10, 100, 1000, etc. Understanding place value and decimal places. Rounding decimals to the nearest whole number or a given number of decimal places. Rounding numbers to a given number of significant figures.	
Using the equivalences between decimals, fractions, ratios and percentages. Converting numbers from one form into another. Ordering and comparing whole numbers, decimals, fractions and percentages. Understanding and using directed numbers, including ordering directed numbers.	
Understanding number relationships and methods of calculation	
Using the common properties of numbers, including odd, even, multiples, factors, primes. Expressing numbers as the product of their prime factors. Least common multiple and highest common factor. Finding the LCM and HCF of numbers written as the product of their prime factors. Using the terms square, square root, cube, cube root and reciprocal. The use of index notation for zero, positive and negative integral indices. The use of index notation for positive unit fractional indices. Interpreting numbers written in standard form in the context of a calculator display. Writing whole numbers in index form. Using the rules of indices. Expressing and using numbers in standard form with positive and negative powers of 10.	

Intermediate tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Using the facilities of a calculator, including the constant function, memory and brackets, to plan a calculation and evaluate expressions. Using addition, subtraction, multiplication, division, square, square root, power, root, constant, memory, brackets and appropriate statistical functions. Knowing how a calculator orders its operations. (Candidates will not be expected to list the key depressions that they have made.) Using calculators effectively and efficiently. Reading a calculator display correct to a specified number of decimal places or significant figures. Using appropriate trigonometric functions on a calculator.	Trigonometry (up to right-angled triangles) can be assessed on GCSE Mathematics - Numeracy and on GCSE Mathematics.
Understanding and using number operations and the relationships between them, including inverse operations and the hierarchy of operations. Addition, subtraction, multiplication and division of whole numbers, decimals, fractions and negative numbers. Finding a fraction or percentage of a quantity. Expressing one number as a fraction or percentage of another. Calculating fractional and percentage changes (increase and decrease), including the use of multipliers. Repeated proportional changes; appreciation and depreciation. Calculating using ratios in a variety of situations; proportional division. Direct and inverse proportion. The use of a non-calculator method to multiply and divide whole numbers up to and including the case of multiplication and division of a three-digit number by a two-digit number.	On GCSE Mathematics - Numeracy, direct and inverse proportion will be assessed through number questions. Note that the algebraic aspect of direct and inverse proportion is assessed on GCSE Mathematics (Higher tier) only. See exemplification in this teachers' guide.
Recognising that recurring decimals are exact fractions, and that some exact fractions are recurring decimals.	
Estimating and approximating solutions to numerical calculations. Using estimation in multiplication and division problems with whole numbers to obtain approximate answers, e.g. by first rounding the numbers involved to 1 significant figure. Candidates must show sufficient working in order to demonstrate how they have obtained their estimate.	

Intermediate tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Solving numerical problems	
Interpretation and use of mathematical information presented in written or visual form when solving problems, e.g. TV programme schedules, bus/rail timetables, distance charts, holiday booking information. Money: The basic principles of personal and household finance, including fuel and other bills, hire purchase, discount, VAT, taxation, best buys, wages and salaries, loan repayments, mortgages, budgeting, exchange rates and commissions. Simple and compound interest, including the use of efficient calculation methods. Profit and loss. Finding the original quantity given the result of a proportional change. Foreign currencies and exchange rates. Carrying out calculations relating to enterprise, saving and borrowing, investing, appreciation and depreciation.	See the section in this teachers' guide on vocabulary of finance.
Giving solutions in the context of a problem, selecting an appropriate degree of accuracy, interpreting the display on a calculator, and recognising limitations on the accuracy of data and measurements. Rounding an answer to a reasonable degree of accuracy in the light of the context. Interpreting the display on a calculator. Knowing whether to round up or down as appropriate. Recognising that measurement is approximate and that a measurement expressed to a given unit is in possible error of half a unit. The upper and lower bounds of numbers expressed to a given degree of accuracy. Calculating the upper and lower bounds in the addition and subtraction of numbers expressed to a given degree of accuracy.	
Understanding and using Venn diagrams to solve problems.	

Venn diagrams is a new topic in these specifications. See the section on new content. This also covers set notation, and what is needed at each tier.

Intermediate tier – Algebra

GCSE Mathematics – Numeracy and GCSE Mathematics	**GCSE Mathematics only***
Understanding and using functional relationships	
Recognition, description and continuation of patterns in number. Description, in words and symbols, of the rule for the next term of a sequence.	Finding the *n*th term of a sequence where the rule is linear or quadratic. Generating linear and non-linear sequences given the *n*th term rule.
Construction and interpretation of conversion graphs. Construction and interpretation of travel graphs. Construction and interpretation of graphs that describe real-life situations. Interpretation of graphical representation used in the media, recognising that some graphs may be misleading. Recognising and interpreting graphs that illustrate direct and inverse proportion.	
Using coordinates in 4 quadrants. Drawing, interpreting, recognising and sketching the graphs of $x = a, y = b, y = ax + b.$ The gradients of parallel lines. Equations of perpendicular lines is new to this specification. See the section on new content in this teachers' guide.	Identifying the equation of lines parallel or perpendicular to a given line, to satisfy given conditions. Knowledge and use of the form $y = mx + c$ to represent a straight line where m is the gradient of the line, and c is the value of the y-intercept. Drawing, interpretation, recognition and sketching the graphs of $y = ax^2 + b.$ Drawing and interpretation of graphs of $y = ax^2 + bx + c.$ Drawing and interpreting graphs when y is given implicitly in terms of x.

Intermediate tier – Algebra

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using equations and formulae	
Substitution of positive and negative whole numbers, fractions and decimals into simple formulae expressed in words or in symbols. Understanding the basic conventions of algebra. Formation and simplification of expressions involving sums, differences, products and powers. Collection of like terms. Expansion of $a(bx + c)$, where a, b and c are integers. Formation and manipulation of linear equations. Changing the subject of a formula when the subject appears in one term.	Extraction of common factors. Formation and manipulation of simple linear inequalities. Multiplication of two linear expressions; expansion of $(ax + by)(cx + dy)$ and $(ax + by)^2$, where a, b, c, d are integers. Factorisation of quadratic expressions of the form $x^2 + ax + b$.
The solution of linear equations with whole number coefficients in solving problems set in real-life contexts.	Solution of linear equations and linear inequalities with whole number and fractional coefficients. The formation and solution of two simultaneous linear equations with whole number coefficients by graphical and algebraic methods in solving problems set in real-life contexts. Solution by factorisation and graphical methods of quadratic equations of the form $x^2 + ax + b = 0$. Solution of a range of cubic equations by trial and improvement methods, justifying the accuracy of the solution.
	Distinguishing in meaning between equations, formulae and expressions.

The basic conventions of algebra, collecting terms, expanding brackets and solving equations can all be assessed on GCSE Mathematics - Numeracy. Procedural questions (out of context) involving this algebra will be assessed on GCSE Mathematics.

Intermediate tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using properties of shape	
The geometrical terms: point, line, plane, parallel, right angle, clockwise and anticlockwise turns, perpendicular, horizontal, vertical, acute, obtuse and reflex angles, face, edge and vertex. Vocabulary of triangles, quadrilaterals and circles: isosceles, equilateral, scalene, exterior/interior angle, diagonal, square, rectangle, parallelogram, rhombus, kite, trapezium, polygon, pentagon, hexagon, radius, diameter, tangent, circumference, chord, arc, sector, segment. Simple solid figures: cube, cuboid, cylinder, prism, pyramid, cone, sphere, tetrahedron. Interpretation and drawing of nets. Using and drawing 2-D representations of 3-D shapes, including the use of isometric paper.	The vocabulary of shapes is included in GCSE Mathematics - Numeracy, so that the words and concepts can be used in this GCSE, as well as in the Mathematics GCSE.
Accurate use of ruler, pair of compasses and protractor. (Lengths accurate to 2mm and angles accurate to 2°.) Bisecting a given line, bisecting a given angle. Constructing the perpendicular from a point to a line. Essential properties of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus; classify quadrilaterals by their geometric properties. Constructing 2-D shapes from given information and drawing plans and elevations of any 3-D solid.	Use of ruler and pair of compasses to do constructions. Construction of triangles, quadrilaterals and circles. Constructing angles of 60°, 30°, 90° and 45°. The identification of congruent shapes.
Candidates will be expected to bisect using a pair of compasses and a ruler in GCSE Mathematics - Numeracy, but constructing angles and shapes can only appear on GCSE Mathematics.	Simple description of symmetry in terms of reflection in a line/plane or rotation about a point. Order of rotational symmetry.

Intermediate tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Angles at a point. Angles at a point on a straight line. Opposite angles at a vertex. Parallel lines. Corresponding, alternate and interior angles. Angle properties of triangles. Using the fact that the angle sum of a triangle is 180°.	Using the fact that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices. Using angle properties of equilateral, isosceles and right-angled triangles; understand congruence; explain why the angle sum of any quadrilateral is 360°. Regular and irregular polygons. Sum of the interior and sum of the exterior angles of a polygon.
Using Pythagoras' theorem in 2-D, including reverse problems.	
Using trigonometric relationships in right-angled triangles to solve problems, including those involving bearings and angles of elevation and depression. Calculating a side or an angle of a right-angled triangle in 2-D.	
	Using angle and tangent properties of circles. Understanding that the tangent at any point on a circle is perpendicular to the radius at that point. Using the facts that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference, that the angle subtended at the circumference by a semicircle is a right angle, that angles in the same segment are equal, and that opposite angles of a cyclic quadrilateral sum to 180°. Understanding and using the fact that tangents from an external point are equal in length.

Trigonometry in right-angled triangles can be assessed in GCSE Mathematics - Numeracy, as well as in GCSE Mathematics.

Most of the circle theorems can be assessed on Intermediate tier. The alternate segment theorem and algebraic proofs can only be assessed on Higher tier. Candidates will not be expected to prove the circle theorems.

Intermediate tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using properties of position, movement and transformation	
	Finding the coordinates of points identified by geometrical information, for example, finding the coordinates of the mid-point of the line segment *AB*, given points *A* and *B*; finding the coordinates of the fourth vertex of a parallelogram, given the coordinates of the other three vertices. Location determined by distance from a given point and angle made with a given line.
Using the knowledge that, for two similar 2-D or 3-D shapes, one is an enlargement of the other. Using the knowledge that, in similar shapes, corresponding dimensions are in the same ratio.	Transformations, including: • Reflection • Rotation through 90°, 180°, 270°. Clockwise or anticlockwise rotations; centre of rotation • Enlargement with positive, fractional scale factors • Translation; description of translations using column vectors. Candidates will be expected to draw the image of a shape under transformation. Questions may involve two successive transformations.
Solving problems in the context of tiling patterns and tessellation.	
Using and interpreting maps. Interpretation and construction of scale drawings. Scales may be written in the form 1 cm represents 5 m, or 1:500. Use of bearings. (Three figure bearings will be used e.g. 065°, 237°.)	
Constructing the locus of a point which moves such that it satisfies certain conditions, for example, (i) a given distance from a fixed point or line, (ii) equidistant from two fixed points or lines. Solving problems involving intersecting loci in two dimensions. Questions on loci may involve inequalities.	

Questions involving the centre of enlargement can appear at Foundation, Intermediate or Higher tiers.

Here is an example of one statement that covers the Foundation and Intermediate tiers. It's important to look at the content for the specific tier you are teaching.

Describing translations as column vectors is new to this specification. See the section on new content in this teachers' guide.

Intermediate tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	**GCSE Mathematics only***
Understanding and using measures	
Standard metric units of length, mass and capacity. The standard units of time; the 12- and 24- hour clock. (The notation for the 12- and 24- hour clock will be 1:30 p.m. and 13:30.) Knowledge and use of the relationship between metric units of length, mass, capacity, area and volume. Making sensible estimates of measurements in everyday situations, recognising the appropriateness of units in different contexts. Conversion between the following metric and Imperial units: km - miles; cm, m - inches, feet; kg - lb; litres - pints, gallons. Candidates will be expected to know the following approximate equivalences: 8km ≈ 5 miles, 1kg ≈ 2·2 lb, 1 litre ≈ 1·75 pints	Candidates will need to know these metric to Imperial conversions. Any others will be given in the examination papers.
Reading and interpreting scales, including decimal scales.	
Distinguishing between formulae for length, area and volume by considering dimensions. Using compound measures including speed, density and population density. Using compound measures such as m/s, km/h, mph, mpg, kg/m^3, g/cm^3, population per km^2.	Dimensional analysis is new to these specifications. See the section on new content in this teachers' guide.
Estimating of the area of an irregular shape drawn on a square grid. Calculating: - perimeter and area of a square, rectangle, triangle, parallelogram, trapezium, circle, semicircle and composite shapes. - surface area, cross-sectional area and volume of cubes, cuboids, prisms, cylinders and composite solids.	Population density is a new aspect of density in these specifications. See the section on new content in this teachers' guide.

Intermediate tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using the statistical problem solving process: specifying the problem/planning; collecting, processing and representing data; interpreting and discussing results.	
Specifying the problem and planning	
Specifying and testing hypotheses, taking account of the limitations of the data available. Testing an hypothesis such as 'Girls tend to do better than boys in biology tests'. Specifying the data needed and considering potential sampling methods. Sampling systematically. Designing and criticising questions for a questionnaire, including notions of fairness and bias. Considering the effect of sample size and other factors that affect the reliability of conclusions drawn.	Sampling is a new topic in these specifications. See the section on new content in this teachers' guide.
Processing, representing and interpreting data	
Sorting, classification and tabulation of qualitative (categorical) data, discrete or continuous quantitative data. Grouping of discrete or continuous data into class intervals of equal or unequal widths. Understanding and using tallying methods.	
Constructing and interpreting pictograms, bar charts and pie charts for qualitative data. Constructing and interpreting vertical line diagrams for discrete data. Constructing line graphs for the values of a variable at different points in time; understanding that intermediate values in a line graph may or may not have meaning. Constructing and interpreting grouped frequency diagrams and frequency polygons. Temperature charts. Constructing and interpreting scatter diagrams for data on paired variables. Constructing and interpreting cumulative frequency tables and diagrams using the upper boundaries of the class intervals.	

Intermediate tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Selecting and using an appropriate measure of central tendency. Mean, median and mode for a discrete (ungrouped) frequency distribution. Estimates for the median and mean of grouped frequency distributions. Comparison of two distributions using one measure of central tendency (i.e. the mean or the median) and/or one measure of spread. Modal category for qualitative data. Modal class for grouped data. Estimating the median from a cumulative frequency diagram. Selecting and calculating or estimating appropriate measures of spread, including the range and interquartile range applied to discrete, grouped and continuous data. Producing and using box-and-whisker plots to compare distributions. Drawing 'by eye' a line of 'best fit' on a scatter diagram, understanding and interpreting what this represents. [In questions where the mean point has been given, calculated or plotted, candidates will be expected to draw the line of 'best fit' through that point.]	Box-and-whisker plots are new to these specifications. See the section on new content in this teachers' guide.
Discussing results	
Recognising that graphs may be misleading. Looking at data to find patterns and exceptions. Drawing inferences and conclusions from summary measures and data representations, relating results back to the original problem. Drawing of conclusions from scatter diagrams; using terms such as positive correlation, negative correlation, little or no correlation. Appreciating that correlation does not imply causality.	

Intermediate tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Estimating and calculating the probabilities of events	
Understanding and using the vocabulary of probability, including notions of uncertainty and risk. The terms 'fair', 'evens', 'certain', 'likely', 'unlikely ' and 'impossible'.	Understanding and using the probability scale from 0 to 1. Use of: the probability of an event not occurring is one minus the probability that it occurs. (Probabilities must be expressed as fractions, decimals or percentages.)
	Estimating the probability of an event as the proportion of times it has occurred. Relative frequency. An understanding of the long-term stability of relative frequency is expected. Graphical representation of relative frequency against the number of trials. Calculating theoretical probabilities based on equally likely outcomes. Estimating probabilities based on experimental evidence. Comparing an estimated probability from experimental results with a theoretical probability.
Venn diagrams and accompanying set notation is explained in more detail in the new content section of this teachers' guide. Note that any Venn diagram question that assesses probability can only appear on GCSE Mathematics.	Identifying all the outcomes of a combination of two experiments, *e.g. throwing two dice;* use tabulation, tree diagrams, Venn diagrams, or other diagrammatic representations of compound events
	Knowledge that the total probability of all the possible outcomes of an experiment is 1. Recognising the conditions when the addition of probabilities for mutually exclusive events and the multiplication of probabilities for two independent events apply, and making the appropriate calculations. If *A* and *B* are mutually exclusive, then the probability of *A* or *B* occurring is $P(A) + P(B)$. If *A* and *B* are independent events, the probability of *A* and *B* occurring is $P(A) \times P(B)$.

Content in standard text:
Even though this content is included at the Higher and Intermediate tiers, it is expected that candidates will be confident and competent in this content at this level. This content can be assessed implicitly at higher and intermediate tier but we wouldn't assess this content directly.

Higher tier

Foundation tier content is in standard text.
Intermediate tier content which is in addition to foundation tier content is in underlined text.
Higher tier content which is in addition to intermediate tier content is in **bold** text.

**Candidates entered for GCSE Mathematics will be expected to be familiar with the knowledge, skills and understanding implicit in GCSE Mathematics – Numeracy.*

Higher tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding number and place value	
Reading and writing whole numbers of any magnitude expressed in figures or words. Rounding whole numbers to the nearest 10, 100, 1000, etc. Understanding place value and decimal places. Rounding decimals to the nearest whole number or a given number of decimal places. Rounding numbers to a given number of significant figures.	
Using the equivalences between decimals, fractions, ratios and percentages. Converting numbers from one form into another. Ordering and comparing whole numbers, decimals, fractions and percentages. Understanding and using directed numbers, including ordering directed numbers.	
Understanding number relationships and methods of calculation	
Using the common properties of numbers, including odd, even, multiples, factors, primes. Expressing numbers as the product of their prime factors. Least common multiple and highest common factor. Finding the LCM and HCF of numbers written as the product of their prime factors. Using the terms square, square root, cube, cube root and reciprocal. The use of index notation for zero, positive and negative integral indices. The use of index notation for positive unit fractional and **other fractional** indices. Interpreting numbers written in standard form in the context of a calculator display. Writing whole numbers in index form. Using the rules of indices. Expressing and using numbers in standard form with positive and negative powers of 10.	

Higher tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Using the facilities of a calculator, including the constant function, memory and brackets, to plan a calculation and evaluate expressions. Using addition, subtraction, multiplication, division, square, square root, power, root, constant, memory, brackets and appropriate statistical functions. Knowing how a calculator orders its operations. (Candidates will not be expected to list the key depressions that they have made.) Using calculators effectively and efficiently. Reading a calculator display correct to a specified number of decimal places or significant figures. Using appropriate trigonometric functions on a calculator.	Trigonometry (up to right-angled triangles) can be assessed on GCSE Mathematics - Numeracy and on GCSE Mathematics.
Understanding and using number operations and the relationships between them, including inverse operations and the hierarchy of operations. Addition, subtraction, multiplication and division of whole numbers, decimals, fractions and negative numbers. Finding a fraction or percentage of a quantity. Expressing one number as a fraction or percentage of another. Calculating fractional and percentage changes (increase and decrease), including the use of multipliers. Repeated proportional changes; appreciation and depreciation. Calculating using ratios in a variety of situations; proportional division. Direct and inverse proportion. The use of a non-calculator method to multiply and divide whole numbers up to and including the case of multiplication and division of a three-digit number by a two-digit number.	On GCSE Mathematics - Numeracy, direct and inverse proportion will be assessed through number questions. Note that the algebraic aspect of direct and inverse proportion is assessed on GCSE Mathematics only. See exemplification in this teachers' guide.
Estimating and approximating solutions to numerical calculations. Using estimation in multiplication and division problems with whole numbers to obtain approximate answers, e.g. by first rounding the numbers involved to 1 significant figure. Candidates must show sufficient working in order to demonstrate how they have obtained their estimate.	

Higher tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Recognising that recurring decimals are exact fractions, and that some exact fractions are recurring decimals. **Converting recurring decimals to fractional form.** **Distinguishing between rational and irrational numbers.** **Manipulating surds; using surds and π in exact calculations.** **Simplifying numerical expressions involving surds,** **excluding the rationalisation of the denominator of a fraction such as** $\frac{1}{(2-\sqrt{3})}$.	Surds (as a topic) can be assessed on either GCSE, but questions set on the Mathematics - Numeracy paper will be set in context, and will not be the procedural questions involving simplifying surds.
Solving numerical problems	
Interpretation and use of mathematical information presented in written or visual form when solving problems, e.g. TV programme schedules, bus/rail timetables, distance charts, holiday booking information. Money: The basic principles of personal and household finance, including fuel and other bills, hire purchase, discount, VAT, taxation, best buys, wages and salaries, loan repayments, mortgages, budgeting, exchange rates and commissions. Simple and compound interest, including the use of efficient calculation methods. Profit and loss. Finding the original quantity given the result of a proportional change. Foreign currencies and exchange rates. Carrying out calculations relating to enterprise, saving and borrowing, investing, appreciation and depreciation and **understanding annual rates, e.g. AER, APR.**	See the section in this teachers' guide on vocabulary of finance. AER/APR is a new topic for these specifications. See the section on new content in this teachers' guide. The AER formula does not need to be learnt. It will be included on the formula page of each examination paper (at Higher tier).
Giving solutions in the context of a problem, selecting an appropriate degree of accuracy, interpreting the display on a calculator, and recognising limitations on the accuracy of data and measurements. Rounding an answer to a reasonable degree of accuracy in the light of the context. Interpreting the display on a calculator. Knowing whether to round up or down as appropriate.	

Higher tier – Number

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Recognising that measurement is approximate and that a measurement expressed to a given unit is in possible error of half a unit. The upper and lower bounds of numbers expressed to a given degree of accuracy. Calculating the upper and lower bounds in the addition and subtraction of numbers expressed to a given degree of accuracy. **Calculating the upper and lower bounds in calculations involving multiplication and division of numbers expressed to given degrees of accuracy.**	Venn diagrams is a new topic in these specifications. See the section on new content. This also covers set notation, and what is needed at each tier.
Understanding and using Venn diagrams to solve problems.	

Higher Tier - Algebra

GCSE Mathematics – Numeracy and GCSE Mathematics	**GCSE Mathematics only***
Understanding and using functional relationships	
Recognition, description and continuation of patterns in number. Description, in words and symbols, of the rule for the next term of a sequence.	Finding the *n*th term of a sequence where the rule is linear or quadratic. Generating linear and non-linear sequences given the *n*th term rule.
Construction and interpretation of conversion graphs. Construction and interpretation of travel graphs. Construction and interpretation of graphs that describe real-life situations. Interpretation of graphical representation used in the media, recognising that some graphs may be misleading. Recognising and interpreting graphs that illustrate direct and inverse proportion.	
Using coordinates in 4 quadrants. Drawing, interpreting, recognising and sketching the graphs of $x = a$, $y = b$, $y = ax + b$. The gradients of parallel lines. Equations of perpendicular lines is new to this specification. See the section on new content in this teachers' guide.	Identifying the equation of lines parallel or perpendicular to a given line, to satisfy given conditions. Knowledge and use of the form $y = mx + c$ to represent a straight line where m is the gradient of the line, and c is the value of the y-intercept. Drawing, interpretation, recognition and sketching the graphs of $y = ax^2 + b$, $y = \frac{a}{x}$, $y = ax^3$. Drawing and interpretation of graphs of $y = ax^2 + bx + c$, $y = ax^3 + b$. **Drawing and interpretation of graphs of** $y = ax + b + \frac{a}{x}$ **with** x **not equal to 0,** $y = ax^3 + bx^2 + cx + d$, $y = k^x$ **for integer values of** x **and simple positive values of** k**.** Drawing and interpreting graphs when y is given implicitly in terms of x.

Higher Tier - Algebra

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
	Understanding and using function notation. **Interpreting and applying the transformation of functions in the context of their graphical representation, including** **$y = \mathrm{f}(x + a)$, $y = \mathrm{f}(kx)$, $y = k\mathrm{f}(x)$ and $y = \mathrm{f}(x) + a$, applied to $y = \mathrm{f}(x)$.**
Constructing and using tangents to curves to estimate rates of change for non-linear functions, and using appropriate compound measures to express results, including finding velocity in distance-time graphs and acceleration in velocity-time graphs. **Interpreting the meaning of the area under a graph, including the area under velocity-time graphs and graphs in other practical and financial contexts.** **Using the trapezium rule to estimate the area under a curve.**	
Understanding and using equations and formulae	
Substitution of positive and negative whole numbers, fractions and decimals into simple formulae expressed in words or in symbols. Understanding the basic conventions of algebra. Formation and simplification of expressions involving sums, differences, products and powers. Collection of like terms. Expansion of $a(bx + c)$, where a, b and c are integers. Formation and manipulation of linear equations. Changing the subject of a formula when the subject appears in one term.	Extraction of common factors. Formation and manipulation of simple linear inequalities. **Changing the subject of a formula when the subject appears in more than one term.** Multiplication of two linear expressions; expansion of $(ax + by)(cx + dy)$ and $(ax + by)^2$, where a, b, c, d are integers. Factorisation of quadratic expressions of the form $x^2 + ax + b$ **and $ax^2 + bx + c$, including the difference of two squares.** **Formation and manipulation of quadratic equations.** **Constructing and using equations that describe direct and inverse proportion.** **Simplifying algebraic fractions.**

This is Higher tier algebra content that can be assessed on GCSE Mathematics - Numeracy

The basic conventions of algebra, collecting terms, expanding brackets and solving equations can all be assessed on GCSE Mathematics - Numeracy. Procedural questions (out of context) involving this algebra will be assessed on GCSE Mathematics.

Here is where the algebraic aspect of direct and inverse proportion is assessed. (See note on page 29 of this subject content.)

Higher Tier - Algebra

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
The solution of linear equations with whole number coefficients in solving problems set in real-life contexts. Regions given by inequalities can be assessed on Higher tier only.	Solution of linear equations and linear inequalities with whole number and fractional coefficients. **The use of straight line graphs to locate regions given by linear inequalities.** The formation and solution of two simultaneous linear equations with whole number coefficients by graphical and algebraic methods in solving problems set in real-life contexts.. Solution by factorisation and graphical methods of quadratic equations of the form $x^2 + ax + b = 0$. **Solution by factorisation, graphical methods and formula, of quadratic equations of the form $ax^2 + bx + c = 0$, selecting the most appropriate method for the problem concerned.** **Solution of equations involving linear denominators leading to quadratic or linear equations.** Solution of a range of cubic equations by trial and improvement methods, justifying the accuracy of the solution.
	Distinguishing in meaning between equations, formulae, **identities** and expressions.

Higher tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using properties of shape	
The geometrical terms: point, line, plane, parallel, right angle, clockwise and anticlockwise turns, perpendicular, horizontal, vertical, acute, obtuse and reflex angles, face, edge and vertex. Vocabulary of triangles, quadrilaterals and circles: isosceles, equilateral, scalene, exterior/interior angle, diagonal, square, rectangle, parallelogram, rhombus, kite, trapezium, polygon, pentagon, hexagon, radius, diameter, tangent, circumference, chord, arc, sector, segment. Simple solid figures: cube, cuboid, cylinder, prism, pyramid, cone, sphere, tetrahedron. Interpretation and drawing of nets. Using and drawing 2-D representations of 3-D shapes, including the use of isometric paper.	
Accurate use of ruler, pair of compasses and protractor. (Lengths accurate to 2mm and angles accurate to 2°.) Bisecting a given line, bisecting a given angle. Constructing the perpendicular from a point to a line. Essential properties of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus; classify quadrilaterals by their geometric properties. Constructing 2-D shapes from given information and drawing plans and elevations of any 3-D solid.	Use of ruler and pair of compasses to do constructions. Construction of triangles, quadrilaterals and circles. Constructing angles of 60°, 30°, 90° and 45°. The identification of congruent shapes. **Understanding and using SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments.** **Reasons may be required in the solution of problems involving congruent triangles.**
	Simple description of symmetry in terms of reflection in a line/plane or rotation about a point. Order of rotational symmetry.

The vocabulary of shapes is included in GCSE Mathematics - Numeracy, so that the words and concepts can be used in this GCSE, as well as in the Mathematics GCSE.

Candidates will be expected to bisect using a pair of compasses and a ruler in GCSE Mathematics - Numeracy, but constructing angles and shapes can only appear on GCSE Mathematics.

Higher tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Angles at a point. Angles at a point on a straight line. Opposite angles at a vertex. Parallel lines. Corresponding, alternate and interior angles. Angle properties of triangles. Using the fact that the angle sum of a triangle is 180°.	Using the fact that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices. Using angle properties of equilateral, isosceles and right-angled triangles; understand congruence; explain why the angle sum of any quadrilateral is 360°. Regular and irregular polygons. Sum of the interior and sum of the exterior angles of a polygon.
Using Pythagoras' theorem in 2-D **and 3-D,** including reverse problems.	
Using trigonometric relationships in right-angled triangles to solve problems, including those involving bearings and angles of elevation and depression. Calculating a side or an angle of a right-angled triangle in 2-D **and 3-D.**	
Trigonometry in right-angled triangles can be assessed in GCSE Mathematics - Numeracy, as well as in GCSE Mathematics.	**Extending trigonometry to angles of any size.** **The graphs and behaviour of trigonometric functions.** **The application of these to the solution of problems in 2-D or 3-D, including appropriate use of the sine and cosine rules.** **Sketching of trigonometric graphs.** **Using the formula: area of a triangle = $\frac{1}{2}ab\sin C$.**
Trigonometry in non-right-angled triangles can only be assessed in GCSE Mathematics. Most of the circle theorems can be assessed on Intermediate tier. The alternate segment theorem and algebraic proofs can only be assessed on Higher tier. Candidates will not be expected to prove the circle theorems.	Using angle and tangent properties of circles. Understanding that the tangent at any point on a circle is perpendicular to the radius at that point. Using the facts that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference, that the angle subtended at the circumference by a semicircle is a right angle, that angles in the same segment are equal, and that opposite angles of a cyclic quadrilateral sum to 180°. **Using the alternate segment theorem.** Understanding and using the fact that tangents from an external point are equal in length. **Understanding and constructing geometrical proofs using circle theorems.**

Higher tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using properties of position, movement and transformation	
	Finding the coordinates of points identified by geometrical information, for example, finding the coordinates of the mid-point of the line segment *AB*, given points *A* and *B*; finding the coordinates of the fourth vertex of a parallelogram, given the coordinates of the other three vertices. Location determined by distance from a given point and angle made with a given line.
Using the knowledge that, for two similar 2-D or 3-D shapes, one is an enlargement of the other. Using the knowledge that, in similar shapes, corresponding dimensions are in the same ratio. **Using the relationships between the ratios of:** • **lengths and areas of similar 2-D shapes, and** • **lengths, areas and volumes of similar 3-D shapes.**	Transformations, including: • Reflection • Rotation through 90°, 180°, 270°. Clockwise or anticlockwise rotations; centre of rotation • Enlargement with positive, fractional **and negative** scale factors • Translation; description of translations using column vectors. Candidates will be expected to draw the image of a shape under transformation. Questions may involve two successive transformations.
Solving problems in the context of tiling patterns and tessellation.	
Using and interpreting maps. Interpretation and construction of scale drawings. Scales may be written in the form 1 cm represents 5 m, or 1:500. Use of bearings. (Three figure bearings will be used e.g. 065°, 237°.)	
Constructing the locus of a point which moves such that it satisfies certain conditions, for example, (i) a given distance from a fixed point or line, (ii) equidistant from two fixed points or lines. Solving problems involving intersecting loci in two dimensions. Questions on loci may involve inequalities.	

Questions involving the centre of enlargement can appear at Foundation, Intermediate or Higher tiers.

Here is an example of one statement that covers the Foundation, Intermediate and Higher tiers. It's important to look at the content for the specific tier you are teaching.

Describing translations as column vectors is new to this specification. See the section on new content in this teachers' guide.

Higher tier – Geometry and Measure

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using measures	
Standard metric units of length, mass and capacity. The standard units of time; the 12- and 24- hour clock. (The notation for the 12- and 24- hour clock will be 1:30 p.m. and 13:30.) Knowledge and use of the relationship between metric units of length, mass, capacity, area and volume. Making sensible estimates of measurements in everyday situations, recognising the appropriateness of units in different contexts. Conversion between the following metric and Imperial units: km - miles; cm, m - inches, feet; kg - lb; litres - pints, gallons. Candidates will be expected to know the following approximate equivalences: 8km ≈ 5 miles, 1kg ≈ 2·2 lb, 1 litre ≈ 1·75 pints	Candidates will need to know these metric to Imperial conversions. Any others will be given in the examination papers.
Reading and interpreting scales, including decimal scales.	
Distinguishing between formulae for length, area and volume by considering dimensions. Using compound measures including speed, density and population density. Using compound measures such as m/s, km/h, mph, mpg, kg/m^3, g/cm^3, population per km^2	Dimensional analysis is new to these specifications. See the section on new content in this teachers' guide.
Estimating of the area of an irregular shape drawn on a square grid. Calculating: - perimeter and area of a square, rectangle, triangle, parallelogram, trapezium, circle, semicircle and composite shapes. - surface area, cross-sectional area and volume of cubes, cuboids, prisms, cylinders and composite solids.	Population density is a new aspect of density in these specifications. See the section on new content in this teachers' guide.
Lengths of circular arcs. **Perimeters and areas of sectors and segments of circles.** **Surface areas and volumes of spheres, cones, pyramids and compound solids.**	

Higher tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Understanding and using the statistical problem solving process: specifying the problem/planning; collecting, processing and representing data; interpreting and discussing results.	
Specifying the problem and planning	
Specifying and testing hypotheses, taking account of the limitations of the data available. Testing an hypothesis such as 'Girls tend to do better than boys in biology tests'. Specifying the data needed and considering potential sampling methods. Sampling systematically **Working with stratified sampling techniques and defining a random sample.** Designing and criticising questions for a questionnaire, including notions of fairness and bias. Considering the effect of sample size and other factors that affect the reliability of conclusions drawn.	Sampling is a new topic in these specifications. See the section on new content in this teachers' guide.
Processing, representing and interpreting data	
Sorting, classification and tabulation of qualitative (categorical) data, discrete or continuous quantitative data. Grouping of discrete or continuous data into class intervals of equal or unequal widths. Understanding and using tallying methods.	
Constructing and interpreting pictograms, bar charts and pie charts for qualitative data. Constructing and interpreting vertical line diagrams for discrete data. Constructing line graphs for the values of a variable at different points in time; understanding that intermediate values in a line graph may or may not have meaning. Constructing and interpreting grouped frequency diagrams and frequency polygons. Temperature charts. Constructing and interpreting scatter diagrams for data on paired variables. Constructing and interpreting cumulative frequency tables and diagrams using the upper boundaries of the class intervals.	

Higher tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Extending skills in handling data into constructing and interpreting histograms with unequal class widths. **Frequency density.** **Interpreting shapes of histograms representing distributions (with reference to mean and dispersion).**	
Selecting and using an appropriate measure of central tendency. Mean, median and mode for a discrete (ungrouped) frequency distribution. Estimates for the median and mean of grouped frequency distributions. Comparison of two distributions using one measure of central tendency (i.e. the mean or the median) and/or one measure of spread. Modal category for qualitative data. Modal class for grouped data. Estimating the median from a cumulative frequency diagram. Selecting and calculating or estimating appropriate measures of spread, including the range and interquartile range applied to discrete, grouped and continuous data. Producing and using box-and-whisker plots to compare distributions. Drawing 'by eye' a line of 'best fit' on a scatter diagram, understanding and interpreting what this represents. [In questions where the mean point has been given, calculated or plotted, candidates will be expected to draw the line of 'best fit' through that point.]	Box-and-whisker plots are new to these specifications. See the section on new content in this teachers' guide.
Discussing results	
Recognising that graphs may be misleading. Looking at data to find patterns and exceptions. Drawing inferences and conclusions from summary measures and data representations, relating results back to the original problem. Drawing of conclusions from scatter diagrams; using terms such as positive correlation, negative correlation, little or no correlation. Appreciating that correlation does not imply causality.	

Higher tier – Statistics

GCSE Mathematics – Numeracy and GCSE Mathematics	GCSE Mathematics only*
Estimating and calculating the probabilities of events	
Understanding and using the vocabulary of probability, including notions of uncertainty and risk. The terms 'fair', 'evens', 'certain', 'likely', 'unlikely ' and 'impossible'.	Understanding and using the probability scale from 0 to 1. Use of: the probability of an event not occurring is one minus the probability that it occurs. (Probabilities must be expressed as fractions, decimals or percentages.)
	Estimating the probability of an event as the proportion of times it has occurred. Relative frequency. An understanding of the long-term stability of relative frequency is expected. Graphical representation of relative frequency against the number of trials. Calculating theoretical probabilities based on equally likely outcomes. Estimating probabilities based on experimental evidence. Comparing an estimated probability from experimental results with a theoretical probability.
Venn diagrams and accompanying set notation is explained in more detail in the new content section of this teachers' guide. Note that any Venn diagram question that assesses probability can only appear on GCSE Mathematics. →	Identifying all the outcomes of a combination of two experiments, *e.g. throwing two dice;* use tabulation, tree diagrams, Venn diagrams, or other diagrammatic representations of compound events.
	Knowledge that the total probability of all the possible outcomes of an experiment is 1. Recognising the conditions when the addition of probabilities for mutually exclusive events and the multiplication of probabilities for two independent events apply, and making the appropriate calculations. If *A* and *B* are mutually exclusive, then the probability of *A* or *B* occurring is *P(A)* + *P(B)*. If *A* and *B* are independent events, the probability of *A* and *B* occurring is *P(A)* × *P(B)*.
	Understanding when and how to estimate conditional probabilities. **The multiplication law for dependent events.** **Sampling without replacement.**

2. ASSESSMENT OBJECTIVES

The titles of the 3 assessment objectives have not changed significantly from the 2010 Linear and Unitised specifications.
The title of AO2 is slightly different for the two GCSEs. Questions assessing AO2 in GCSE Mathematics won't necessarily be set in contexts, whereas they will be in GCSE Mathematics – Numeracy.
The weighting of each assessment objective has changed, and they are different within each GCSE. (See table below.)
Bullet points have been added to the descriptions. In the main, these are there to add clarity to the main statements. This is the case in AO1 and AO2. However, some bullet points have been added in to AO3 as we will be assessing particular aspects of reasoning, interpreting, communicating and problem solving. A brief explanation of these follows the table.
Note that some aspects of AO3 can be assessed in questions that mainly assess AO1 or AO2.

		Weighting in Mathematics - Numeracy	Weighting in Mathematics
AO1	**Recall and use their knowledge of the prescribed content** • Recall and use mathematical facts and concepts. • Recall and use standard mathematical methods. • Follow direct instructions to solve problems involving routine procedures.	**15% - 25%**	**50% - 60%**
AO2	**Select and apply mathematical methods*** • Select and use the mathematics and resources needed to solve a problem. • Select and apply mathematical methods to solve non-standard or unstructured, multi-step problems. • Make decisions when tackling a given task, for example, choose how to display given information. *GCSE Mathematics – Numeracy: Select and apply mathematical methods in a range of contexts	**50% - 60%**	**10% - 20%**

AO3	**Interpret and analyse problems and generate strategies to solve them** • Devise strategies to solve non-routine or unfamiliar problems, breaking them into smaller, more manageable tasks, where necessary. • Communicate mathematically, using a wide range of mathematical language, notation and symbols to explain reasoning and to express mathematical ideas unambiguously. • Construct arguments and proof using logical deduction. • Interpret findings or solutions in the context of the original problem. • Use inferences and deductions made from mathematical information to draw conclusions. • Reflect on results and evaluate the methods employed.	**20% - 30%**	**25% - 35%**

- Devise strategies to solve non-routine or unfamiliar problems, breaking them into smaller, more manageable tasks, where necessary.

  This is what is assessed in AO3 questions currently.

- Communicate mathematically, using a wide range of mathematical language, notation and symbols to explain reasoning and to express mathematical ideas unambiguously.
- Construct arguments and proof using logical deduction.
- Use inferences and deductions made from mathematical information to draw conclusions.

  These are aspects of explaining, reasoning, interpreting and communicating that are now assessed in AO3.

- Interpret findings or solutions in the context of the original problem.

  This is when a candidate links their answer to a calculation to the original context. It may simply be explaining or indicating what their answer represents in the context of the question.

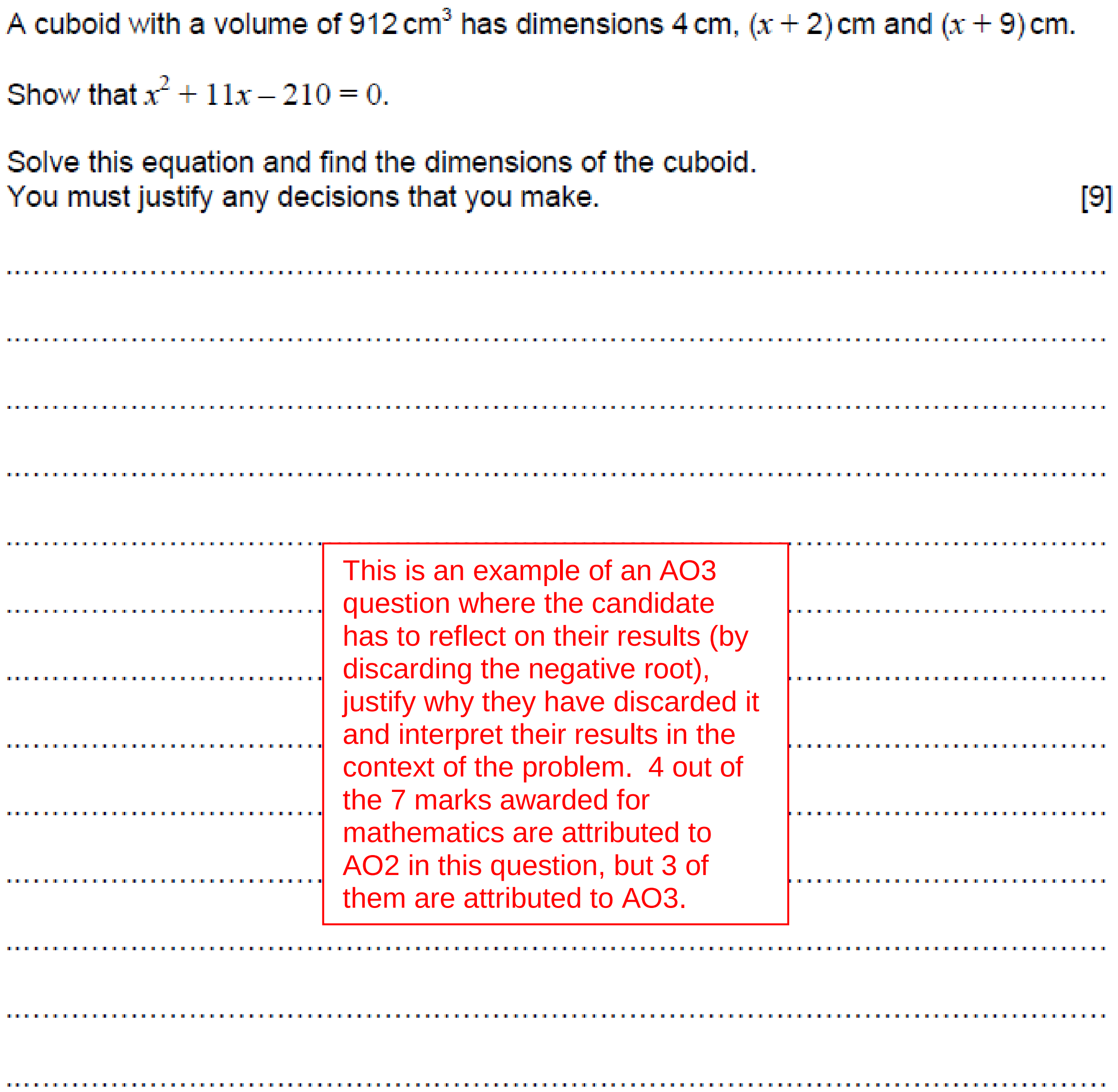
- Reflect on results and evaluate the methods employed.

  Examples of this include, but aren't restricted to:
  - *when a candidate reflects on and evaluates the method used (by themselves or given to them in the question) and comments on its efficiency, for example*
  - *when a candidate obtains a series of results, and only some of them are valid. In which case they would need to discard some, and could be asked to explain why*
  - *when a candidate has to make an assumption to answer a question (or the assumption may be given) and they may be asked to comment on what effect the assumption has had on their answer*

Examples

Important note: These are examples taken from the Specimen Assessment Materials and from SAMs2. This is a selection of questions from these materials that assess AO3. They do not represent every type of AO3 question that could be asked.

1. *SAMs 1 Mathematics Unit 1 Higher*



A cuboid with a volume of 912 cm^3 has dimensions 4 cm, $(x + 2)$ cm and $(x + 9)$ cm.

Show that $x^2 + 11x - 210 = 0$.

Solve this equation and find the dimensions of the cuboid.
You must justify any decisions that you make. [9]

This is an example of an AO3 question where the candidate has to reflect on their results (by discarding the negative root), justify why they have discarded it and interpret their results in the context of the problem. 4 out of the 7 marks awarded for mathematics are attributed to AO2 in this question, but 3 of them are attributed to AO3.

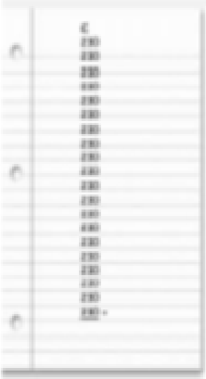
2. *SAMs 1 Mathematics – Numeracy Unit 1 Foundation*

The *Hafod Hotel* has 20 bedrooms.

(a) Andrew is the deputy manager.
He is calculating the cost of buying 20 new single beds.

Single bed £230

Andrew writes out a sum with £230 written 20 times.



Describe a better method that Andrew could use to calculate the cost of 20 beds at £230 each.
Work out the total cost of these 20 beds using your suggested method. [2]

Method:

This is an example of an AO3 question where the candidate has to reflect on a method used that is given in the question, by commenting on its efficiency, and then has to suggest a better method that could be used.

Total cost of 20 beds = £……………

(b) Iona is the hotel manager.
Iona says that 2 single beds are needed for each bedroom, so the hotel needs 40 new single beds not 20.
Describe the quickest way for Andrew to now work out the total cost of the 40 beds.
Write down the total cost of 40 beds. [2]

Method:

Total cost of 40 beds = £……………

3. *SAMs 2 Mathematics - Numeracy Unit 2 Higher*

A cylinder is made of bendable plastic.
Part of a child's toy is made by bending the cylinder to form a ring.
The two circular ends of the cylinder are joined to form the ring.

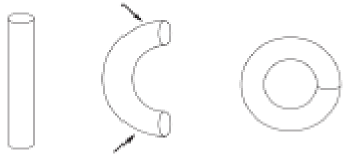


Diagram not drawn to scale

The inner radius of the ring is 9 cm.
The outer radius of the ring is 10 cm.

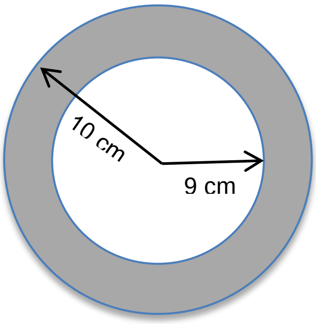


Diagram not drawn to scale

Calculate an approximate value for the volume of the ring.
State and justify what assumptions you have made in your calculations and the impact they have had on your results.

[7]

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This is an example of an AO3 question where the candidate has to make an assumption in order to answer the question. They also have to comment on the effect the assumption has. Again, this is assessing a candidate's ability to 'reflect on results and evaluate the methods employed'.

4. *SAMs 2 Mathematics – Numeracy: Unit 1 Foundation*

Gethin wants to organise a mountain walk in the Brecon Beacons with his 3 friends Chloe, Robert and Martyn during 2015.

He has the following information:
- He (Gethin) can only go on a Sunday;
- Chloe cannot go during the last 4 months of the year;
- Martyn works on the first 3 Sundays of each month;
- Robert cannot go during the school holidays;
- All his friends agree that the months of November, December and January are unsuitable for the walk.

The calendar shown on the opposite page is for 2015.
The school holidays are represented by

▨

What would be the **latest date** that they could all go for the mountain walk?
You may use the calendar provided to show your working.

[5]

This is an example of an AO3 question where the candidate is given information and they have to devise a strategy to solve the problem. Reasoning, communicating and interpreting skills are needed here too.

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JANUARY 2015

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

FEBRUARY 2015

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28

MARCH 2015

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

APRIL 2015

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

MAY 2015

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

JUNE 2015

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

JULY 2015

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

AUGUST 2015

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

SEPTEMBER 2015

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

OCTOBER 2015

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

NOVEMBER 2015

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

DECEMBER 2015

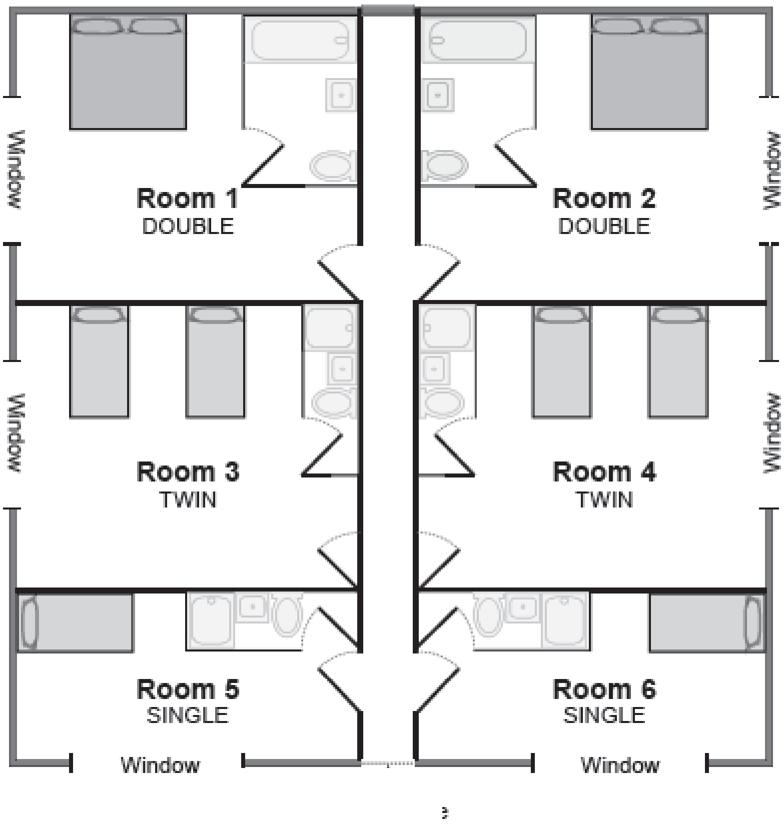
S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

5. *SAMs 2 Mathematics - Numeracy Unit 1 Foundation*

Gwesty Traeth is a guest house and has six bedrooms.

Two of the rooms are described as *Double* (they have a double bed).
Two of the rooms are described as *Twin* (they have two single beds).
Two of the rooms are described as *Single* (they have one single bed).

The diagram below shows a plan of these rooms.



The people listed below have contacted *Gwesty Traeth* requesting rooms for dates in July 2016.

- Sasha and Mia want to share a twin room for the 6th and 7th.
- Mr & Mrs Jones want a double room for the 5th.
- Flavia wants a single room for the 5th and 6th.
- Mr & Mrs Evans want a double room for themselves and a twin room for their sons, Morys and Ifan, to share for the three nights 5th, 6th and 7th.
- Their daughter Heledd will join them on the 6th and 7th, and she requires a single room.
- Mr & Mrs Igorson want a double room for the 6th and 7th.

Use the table below to show who is given which room for each of the dates from the 5th July until the 7th July.
No-one should have to change rooms during their stay.

[4]

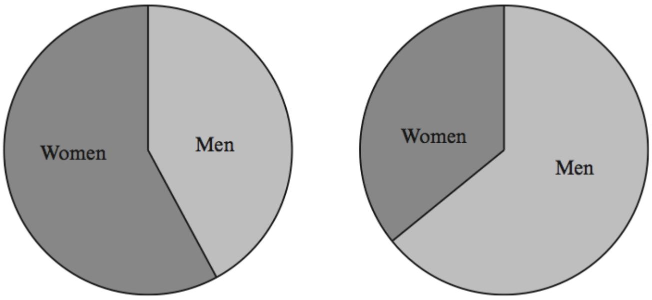
	Room 1	**Room 2**	**Room 3**	**Room 4**	**Room 5**	**Room 6**
5th July						
6th July						
7th July						

This is another example of an AO3 question where the candidate is given information and they have to devise a strategy to solve the problem. Reasoning, communicating and interpreting skills are needed here too.

6. *SAMs 2 Mathematics - Numeracy Unit 1 Intermediate and Higher*

Lucy has been given pie charts showing the number of computers sold by 2 different companies.

RG computers *LF computers*



Lucy says

'More men buy RG computers than LF computers.'

Explain how this could be true. [1]

……………………………………

This is an example of a 1 mark AO3 question where the candidate is asked to explain their reasoning.

……………………………………

……………………………………………………………………………………………

7. *SAMs 2 Mathematics - Numeracy Unit 1 Higher*

A team of examiners has 64 000 examination papers to mark.
It takes each examiner 1 hour to mark approximately 10 papers.

(a) The chief examiner says that a team of 50 examiners could mark all 64 000 papers in 8 days.
What assumption has the chief examiner made?
You must show all your calculations to support your answer.

[4]

This is an example of an AO3 question where the candidate is asked to find the assumption made and to comment on what effect the assumption has. This is a good example of a question where the candidate has to 'reflect on results and evaluate the methods employed'.

(b) Why is the chief examiner's assumption unrealistic?
What effect will this have on the number of days the marking will take?

[2]

8. *SAMs 2 Mathematics - Numeracy Unit 1 Higher*

For a concert, of the 128 adult performers, 52 are male and 76 are female.
Gwen decides to interview a stratified sample of **16 adults** and has exactly 16 copies of the questionnaire ready for them.

Using these numbers, she calculates that she should interview 7 male performers and 10 female performers, making a total of **17 adults**.

Explain how this has happened.

[2]

This is an example of a 2 mark AO3 question where the candidate has to evaluate a method used and has to explain their reasoning.

9. *SAMs 2 Mathematics Unit 1 Foundation and Intermediate*

Sian states,

> 'When a fair coin is tossed and a fair dice is thrown,
> the probability of getting a head and an even number is $\frac{1}{2}$.'

Is Sian correct?
You must show enough working to justify your answer.

[4]

……………………………………………………………………………………………………

This is an example where the candidate has to explain the reasoning behind a response and make a deduction.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

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10. *SAMs 2 Mathematics Unit 2 Foundation and Intermediate*

A bag contains some red, green and black beads.
One bead is selected at random from the bag.

The probability of selecting a green bead from the bag is $\frac{1}{3}$.

Which of the following sets of beads could have been in the bag?
Circle the correct answer.

2 red	3 red	3 red	7 red	5 red
1 green	6 green	3 green	4 green	3 green
1 black	3 black	4 black	1 black	4 black

[1]

This is an example of a 1 mark AO3 question where the candidate has to think of a strategy to find the correct answer.

11. *SAMs 2 Mathematics Unit 1 Intermediate and Higher*

Are the following statements true or false? Circle the correct answer.
You must give a **full explanation** of your decision in each case.

(a)

$a^2 + b^2$ is always an even number when a and b are whole numbers.

[1]

true / false

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

This is an example of a 1 mark AO3 question where the candidate is asked to explain their reasoning.

(b)

$a^2 b^2$ is always an odd number when a and b are odd numbers.

[2]

true / false

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

This is an example of a 2 mark AO3 question where the candidate is asked to explain their reasoning.

3. Summary of new content topics

	Topic	Mathematics-Numeracy	Mathematics only	Tier
Number	AER, APR	✓		Higher
	Venn diagrams	✓		Foundation Intermediate Higher
Algebra	Equations of perpendicular lines		✓	Intermediate Higher
Geometry and measure	Dimensions	✓		Intermediate Higher
	Population density	✓		Intermediate Higher
	Translation (expressed as a vector)		✓	Intermediate Higher
Statistics	Box-and-whisker plots (box plots)	✓		Intermediate Higher
	Sampling (considering potential sampling methods, systematic sampling)	✓		Intermediate Higher
	Sampling (stratified sampling, random sampling)	✓		Higher
	Venn diagrams		✓	Foundation Intermediate Higher

3.1 AER

Specification statement (Higher tier, Mathematics - Numeracy and Mathematics)

> Carrying out calculations relating to enterprise, saving and borrowing, investing, appreciation and depreciation, **and understanding annual rates, e.g. AER, APR.**

Notes

1. AER = annual equivalent rate.

This gives the percentage interest earned in a savings or investment account in one year. It enables **comparison** of rates between different lenders and accounts which pay interest at different frequencies e.g. each month, quarter, 6 months.

Example

A savings account is advertised as paying 4·28% interest on an investment of £100, with interest payments made once every 3 months.

The interest rate is therefore divided by 4 (the number of times it is paid per year) to give 4·28 ÷ 4 = 1·07%.

After the first 3 months, the account is worth £100 × 1·0107 = £101.07.

> **** It would be an easy mistake to assume that the additional amount paid every 3 months is always £1.07 ****

The interest is COMPOUNDED every 3 months.

After 6 months, the account is worth £101.07 × 1·0107 **OR** £100 × $1{\cdot}0107^2$ = £102.15

After 9 months, the account is worth £102.15 × 1·0107 **OR** £100 × $1{\cdot}0107^3$ = £103.24

After 12 months, the account is worth £103.24 × 1·0107 **OR** £100 × $1{\cdot}0107^4$ = £104.35

From the value of the savings afer 12 months, it appears that the AER is 4·35%.

This value could have been calculated more quickly using the formula

$$\left(1+\frac{i}{n}\right)^{n}-1$$

where i is 'the nominal interest rate per annum', in this case 4·28%, and n is 'the number of compunding periods per annum', in this case 12 ÷ 3 = 4.
Then we have

$$\left(1+\frac{0.0428}{4}\right)^{n}-1$$

$$= 1{\cdot}043491.....\ -1$$

$$= 0{\cdot}043491 \quad \text{OR} \quad 4{\cdot}35\%\ (2\ \text{d.p.})$$

It is vital to understand that **compounding** the interest has the effect of ‘increasing’ the percentage interest rate

e.g. 1% compound interest per month for 1 year gives greater interest than 12% as an annual rate.

2. APR = annual percentage rate

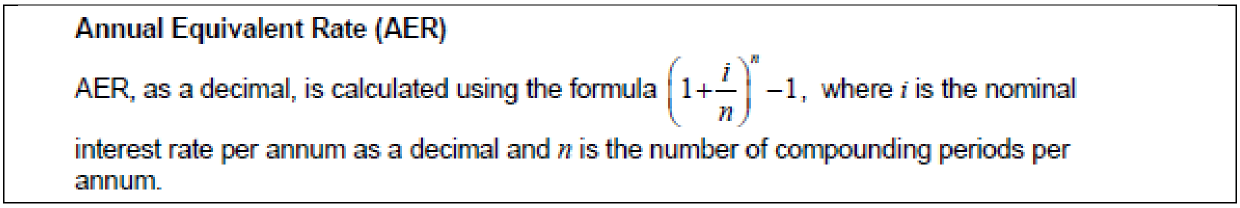
This measures the cost of borrowing money. The calculation includes fees charged by the lender for setting up the loan.

3. EAR = equivalent annual rate

Again, this measures the cost of borrowing money, though this time in the form of an overdraft.

Examples of examination questions on AER

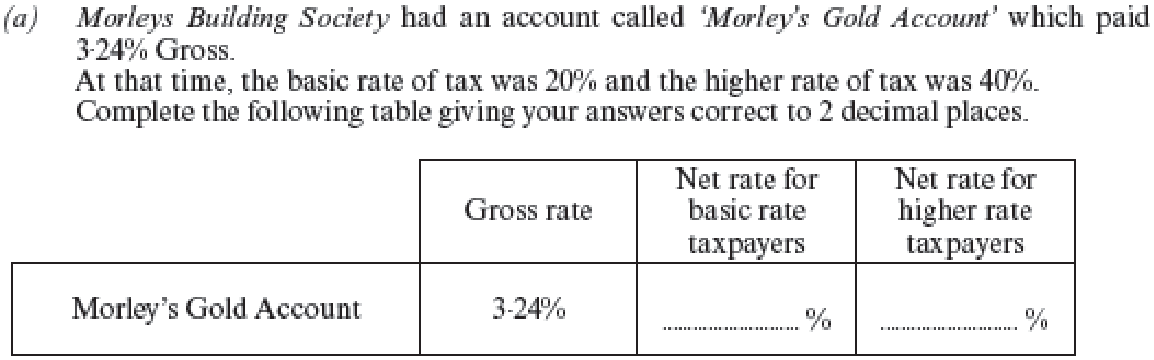
From the formula list given at the beginning of a Higher tier paper:



Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1+\frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.

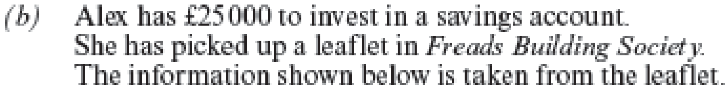
1. *June 13 Applications of Mathematics Unit 2 Higher*



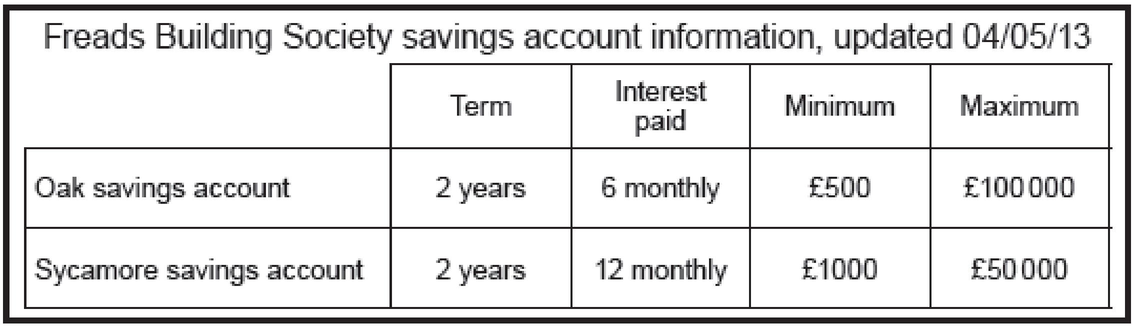
(a) *Morleys Building Society* had an account called *'Morley's Gold Account'* which paid 3·24% Gross.
At that time, the basic rate of tax was 20% and the higher rate of tax was 40%.
Complete the following table giving your answers correct to 2 decimal places.

	Gross rate	Net rate for basic rate taxpayers	Net rate for higher rate taxpayers
Morley's Gold Account	3·24%	.... %	.... %

[4]

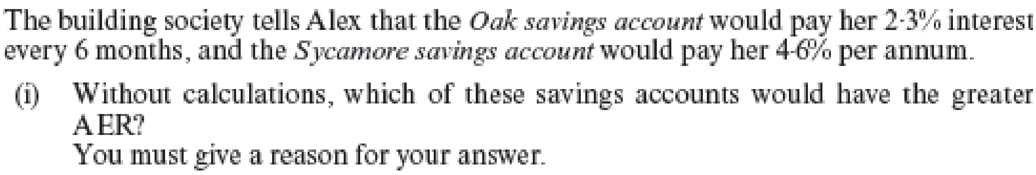


(b) Alex has £25 000 to invest in a savings account.
She has picked up a leaflet in *Freads Building Society*.
The information shown below is taken from the leaflet.



Freads Building Society savings account information, updated 04/05/13

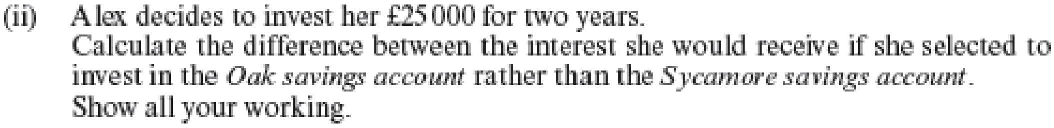
	Term	Interest paid	Minimum	Maximum
Oak savings account	2 years	6 monthly	£500	£100 000
Sycamore savings account	2 years	12 monthly	£1000	£50 000



The building society tells Alex that the *Oak savings account* would pay her 2·3% interest every 6 months, and the *Sycamore savings account* would pay her 4·6% per annum.

(i) Without calculations, which of these savings accounts would have the greater AER?
You must give a reason for your answer.

[1]



(ii) Alex decides to invest her £25 000 for two years.
Calculate the difference between the interest she would receive if she selected to invest in the *Oak savings account* rather than the *Sycamore savings account*.
Show all your working.

[6]

2. *June 2012 Applications of Mathematics Unit 2 Higher*

Adam is interested in opening a savings account at Morris Bank.
The manager of Morris Bank explains to Adam that they have two different savings accounts.
Some details of the accounts are shown below.

Account	Nominal interest rate	AER Annual Equivalent Rate, correct to 2 decimal places
Quarter Back	8·6% p.a., paid quarterly	………………%
Monthly Goal	5·4% p.a., paid monthly	5·54%

(a) (i) In the table above, complete the AER column in the table for the Quarter Back account using the information given below.

[5]

(ii) Explain why AER is used by the bank.

[1]

3. *January 14 Applications of Mathematics Unit 2 Higher*

Haygreen Building Society offers customers a range of savings accounts.

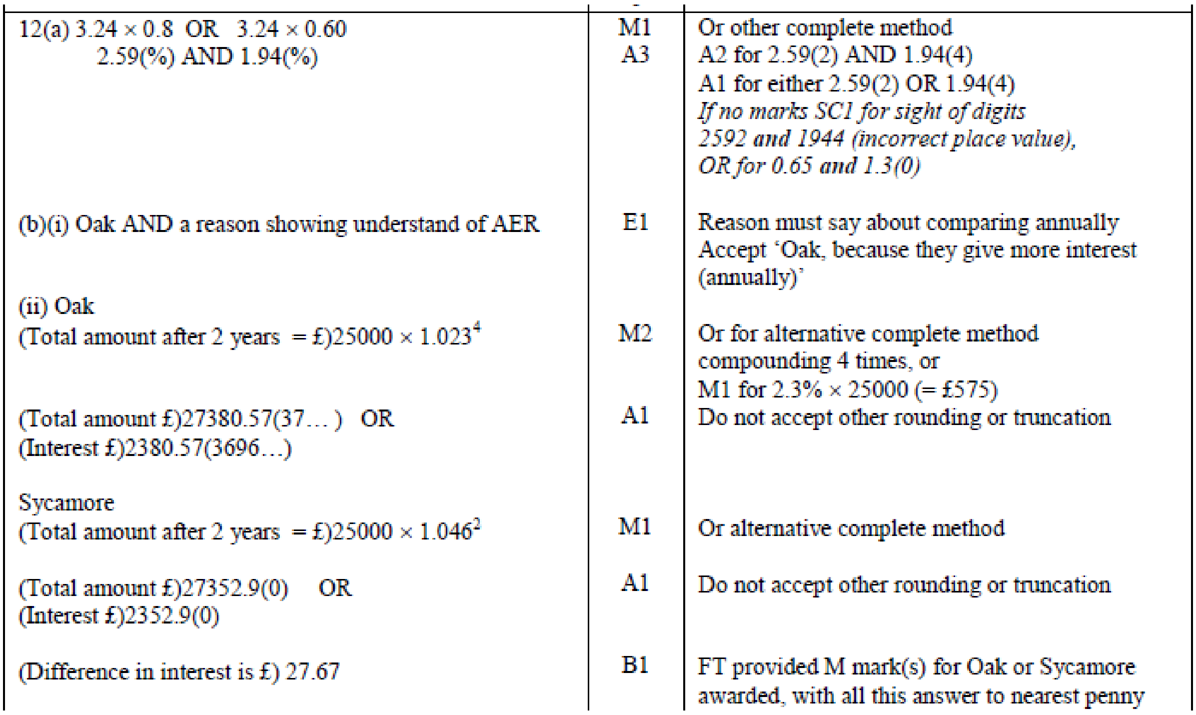
£££££ **Mega Plus** £££££
Gold
Save & Grow

(i) The Gross annual interest rate on the **Mega Plus** savings account is 4·8%, with the interest payable monthly.
Calculate the monthly interest rate payable on the **Mega Plus** savings account. [1]

(ii) Ffion decides to open a **Gold** savings account on the 1st May.
The interest is paid at a rate of 0·3% per month.
She invests £200 in the account.
She leaves the account without withdrawing from or making payments into her account for 5 months.
Calculate the balance that would be shown on Ffion's **Gold** savings account statement after this five-month period. [3]

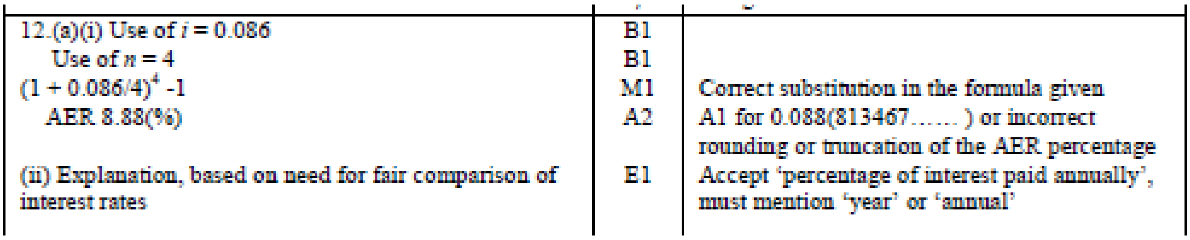
Mark schemes for examination questions on AER

1. *June 13 Applications of Mathematics Unit 2 Higher*



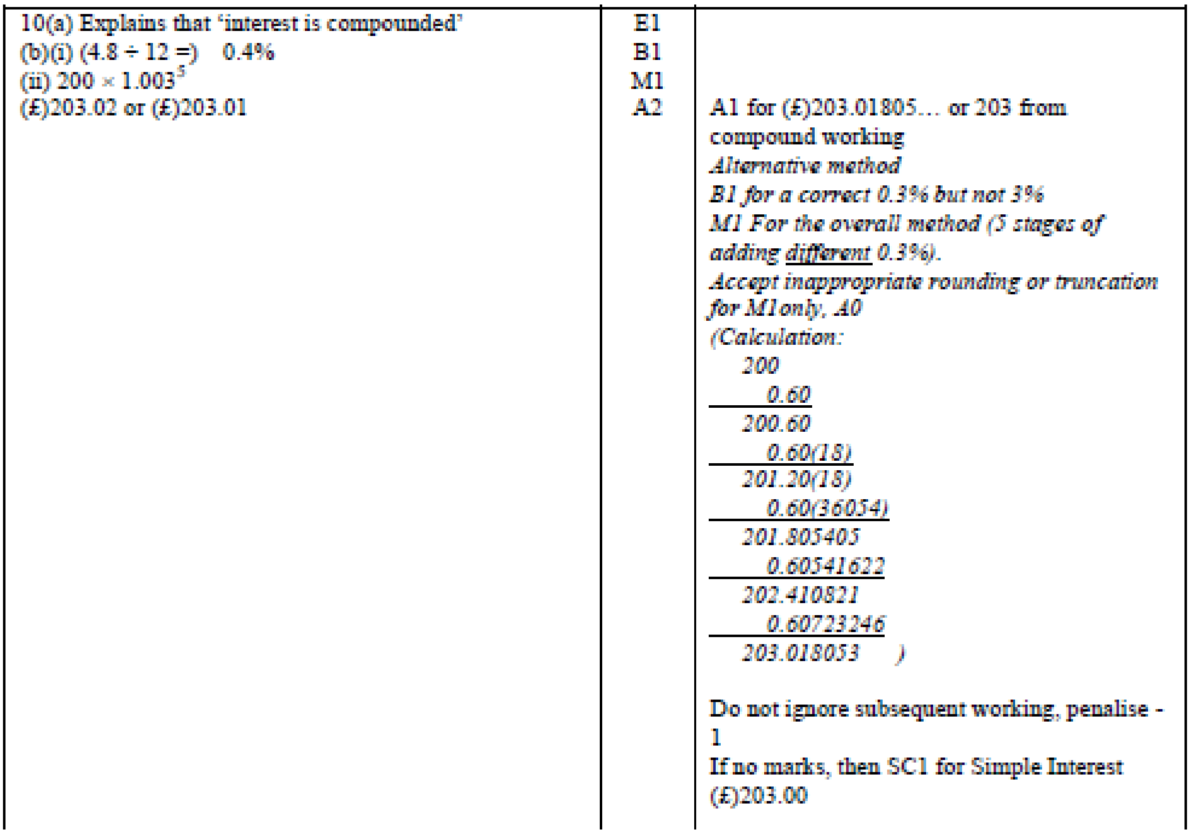
12(a) 3.24 × 0.8 OR 3.24 × 0.60	M1	Or other complete method
2.59(%) AND 1.94(%)	A3	A2 for 2.59(2) AND 1.94(4) A1 for either 2.59(2) OR 1.94(4) *If no marks SC1 for sight of digits 2592 and 1944 (incorrect place value), OR for 0.65 and 1.3(0)*
(b)(i) Oak AND a reason showing understand of AER	E1	Reason must say about comparing annually Accept 'Oak, because they give more interest (annually)'
(ii) Oak (Total amount after 2 years = £)25000 × 1.023^4	M2	Or for alternative complete method compounding 4 times, or M1 for 2.3% × 25000 (= £575)
(Total amount £)27380.57(37…) OR (Interest £)2380.57(3696…)	A1	Do not accept other rounding or truncation
Sycamore (Total amount after 2 years = £)25000 × 1.046^2	M1	Or alternative complete method
(Total amount £)27352.9(0) OR (Interest £)2352.9(0)	A1	Do not accept other rounding or truncation
(Difference in interest is £) 27.67	B1	FT provided M mark(s) for Oak or Sycamore awarded, with all this answer to nearest penny

2. *June 2012 Applications of Mathematics Unit 2 Higher*



12.(a)(i) Use of $i = 0.086$	B1	
Use of $n = 4$	B1	
$(1 + 0.086/4)^4$ -1	M1	Correct substitution in the formula given
AER 8.88(%)	A2	A1 for 0.088(813467……) or incorrect rounding or truncation of the AER percentage
(ii) Explanation, based on need for fair comparison of interest rates	E1	Accept 'percentage of interest paid annually', must mention 'year' or 'annual'

3. *January 14 Applications of Mathematics Unit 2 Higher*



10(a) Explains that 'interest is compounded'	E1	
(b)(i) (4.8 ÷ 12 =) 0.4%	B1	
(ii) 200×1.003^5	M1	
(£)203.02 or (£)203.01	A2	A1 for (£)203.01805… or 203 from compound working *Alternative method* *B1 for a correct 0.3% but not 3%* *M1 For the overall method (5 stages of adding different 0.3%).* *Accept inappropriate rounding or truncation for M1only, A0* *(Calculation:* *200* *0.60* *200.60* *0.60(18)* *201.20(18)* *0.60(36054)* *201.805405* *0.60541622* *202.410821* *0.60723246* *203.018053)* Do not ignore subsequent working, penalise - 1 If no marks, then SC1 for Simple Interest (£)203.00

Further examples of questions can be found on the WJEC website in Unit 2 Higher Applications of Mathematics papers (4362/02) from January 2011 onwards (January and June series).

Worked and marked example on AER

SAMs 2 Mathematics – Numeracy Unit 2 Higher

Dragon Nation Bank is advertising a savings account.

Account	**Nominal interest rate**	**AER** **Annual Equivalent Rate, correct to 2 decimal places**
Dragon Saver	7·6% p.a., paid quarterly	………………. %

(a) Complete the AER entry in the table.

[4]

……………………………………………………………………………………….

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……………………………………………………………………………………….

(b) Explain why AER is used by the bank.

[1]

……………………………………………………………………………………….

……………………………………………………………………………………….

……………………………………………………………………………………….

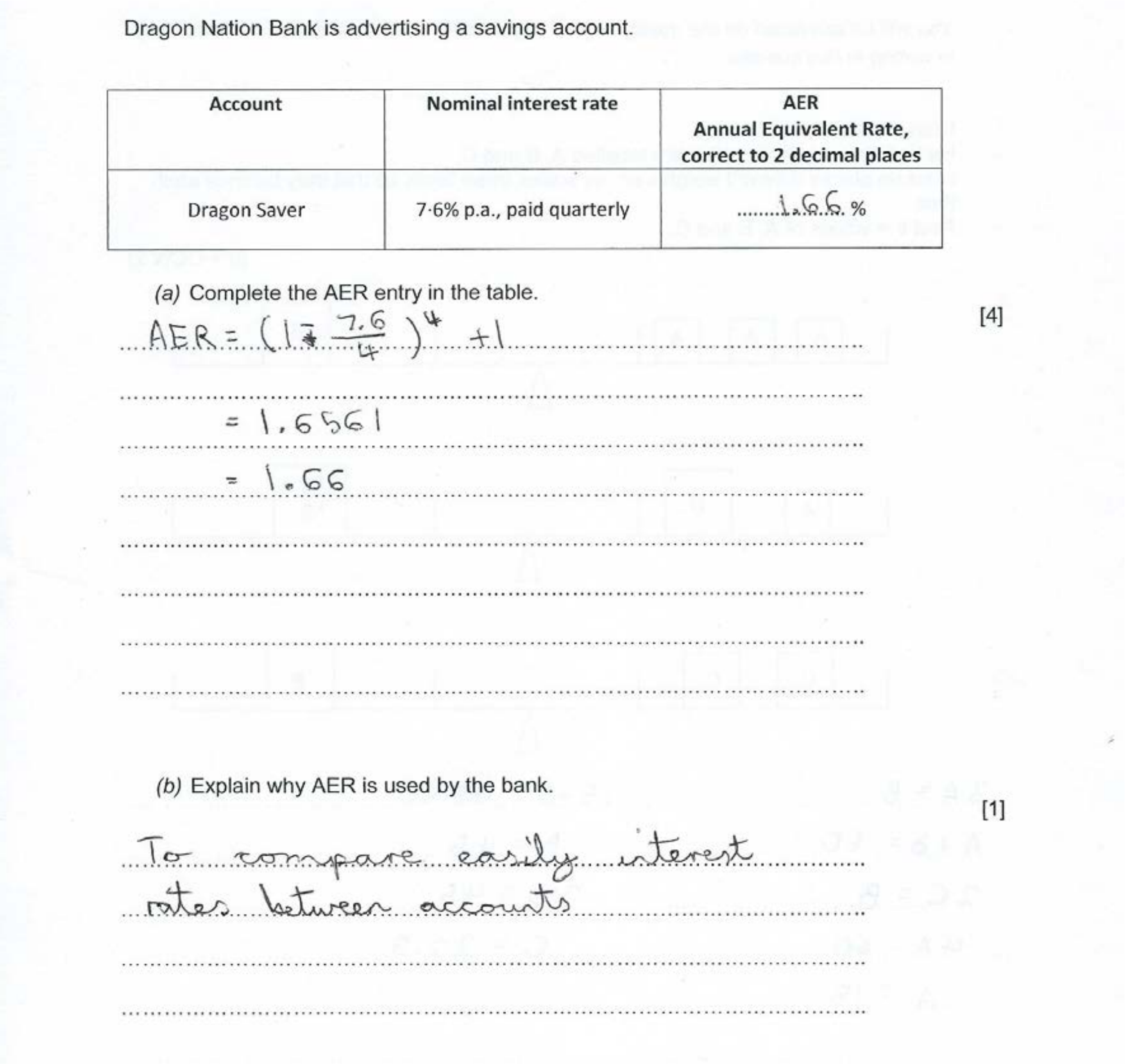
……………………………………………………………………………………….

Mark scheme

11. (a) Use of *i* = 0·076 AND *n* = 4 $(1 + 0{\cdot}076 / 4)^4 - 1$ AER 7·82(%)	 B1 M1 A2	Check table. Correct substitution in the formula. A1 for 0·078(19...) or incorrect rounding or truncation of the AER percentage.
(b) Explanation, based on need for fair comparison of interest rates.	E1 5	Accept 'percentage of interest paid annually'.

Candidate responses

Candidate A



Dragon Nation Bank is advertising a savings account.

Account	Nominal interest rate	AER Annual Equivalent Rate, correct to 2 decimal places
Dragon Saver	7·6% p.a., paid quarterly	1.66 %

(a) Complete the AER entry in the table. [4]

$AER = (1 \mp \frac{7.6}{4})^4 + 1$

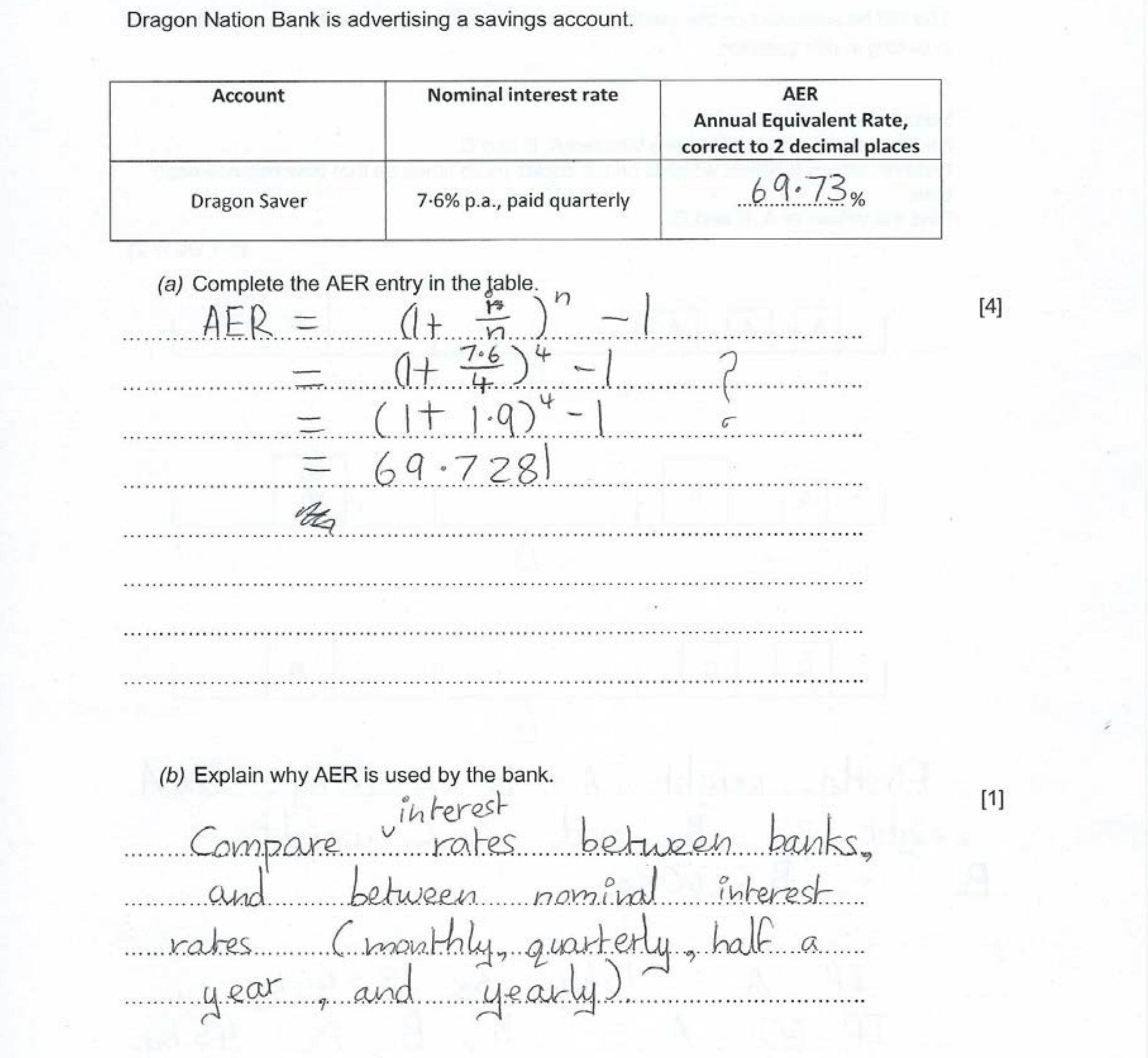
$= 1.6561$

$= 1.66$

(b) Explain why AER is used by the bank. [1]

To compare easily interest rates between accounts

Candidate B



Dragon Nation Bank is advertising a savings account.

Account	Nominal interest rate	AER Annual Equivalent Rate, correct to 2 decimal places
Dragon Saver	7·6% p.a., paid quarterly	69·73%

(a) Complete the AER entry in the table. [4]

$$AER = \left(1+\frac{r}{n}\right)^n - 1$$

$$= \left(1+\frac{7.6}{4}\right)^4 - 1 \quad ?$$

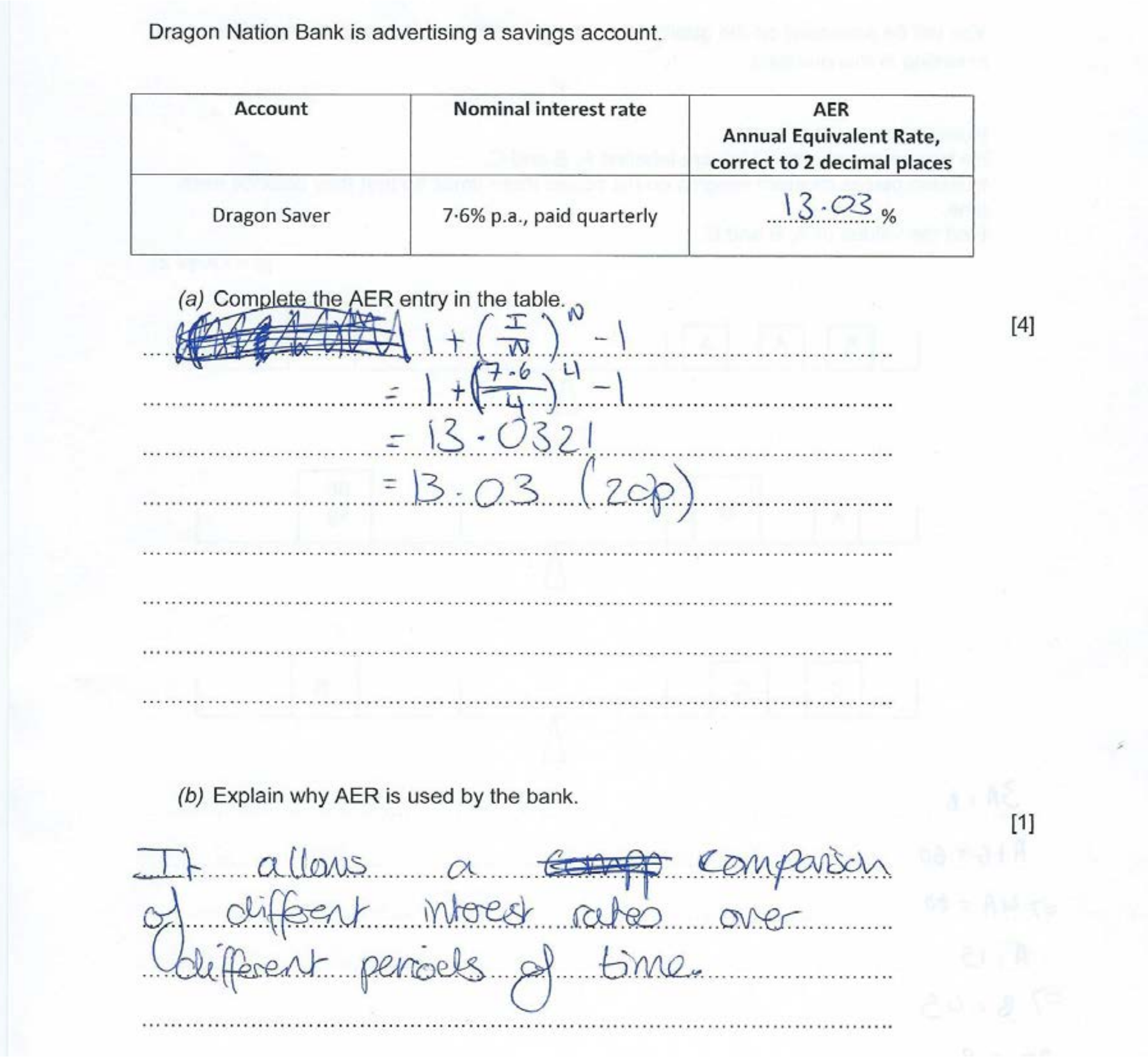
$$= (1+1.9)^4 - 1$$

$$= 69.7281$$

(b) Explain why AER is used by the bank. [1]

Compare interest rates between banks, and between nominal interest rates (monthly, quarterly, half a year, and yearly).

Candidate C



Dragon Nation Bank is advertising a savings account.

Account	Nominal interest rate	AER Annual Equivalent Rate, correct to 2 decimal places
Dragon Saver	7·6% p.a., paid quarterly	13.03 %

(a) Complete the AER entry in the table. [4]

$1+\left(\frac{I}{n}\right)^{n}-1$

$= 1+\left(\frac{7.6}{4}\right)^{4}-1$

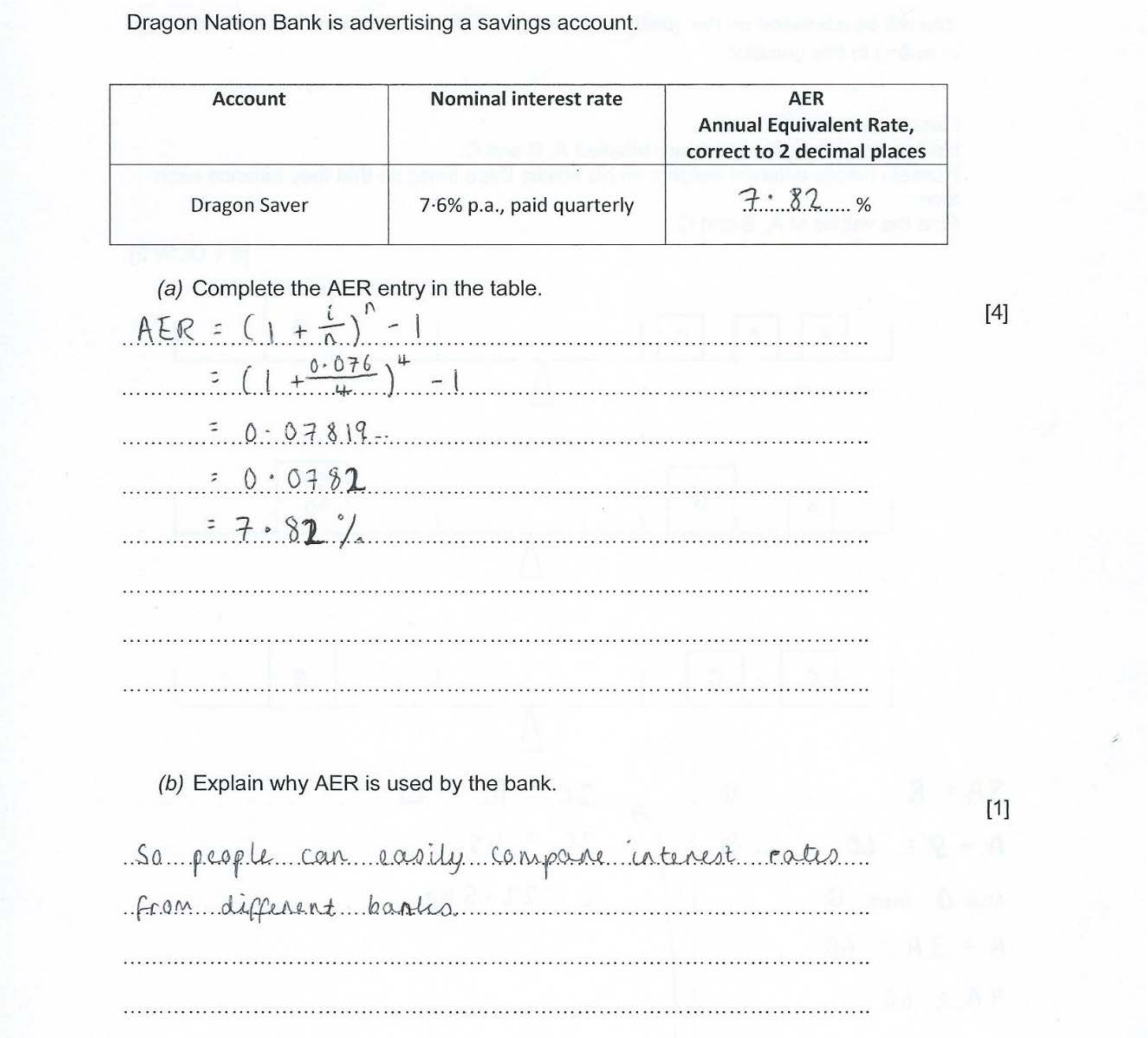
$= 13.0321$

$= 13.03$ (2dp)

(b) Explain why AER is used by the bank. [1]

It allows a ~~compp~~ comparison of different interest rates over different periods of time.

Candidate D



Dragon Nation Bank is advertising a savings account.

Account	Nominal interest rate	AER Annual Equivalent Rate, correct to 2 decimal places
Dragon Saver	7·6% p.a., paid quarterly	7·82 %

(a) Complete the AER entry in the table. [4]

$AER = \left(1 + \frac{i}{n}\right)^n - 1$

$= \left(1 + \frac{0 \cdot 076}{4}\right)^4 - 1$

$= 0 \cdot 07819..$

$= 0 \cdot 0782$

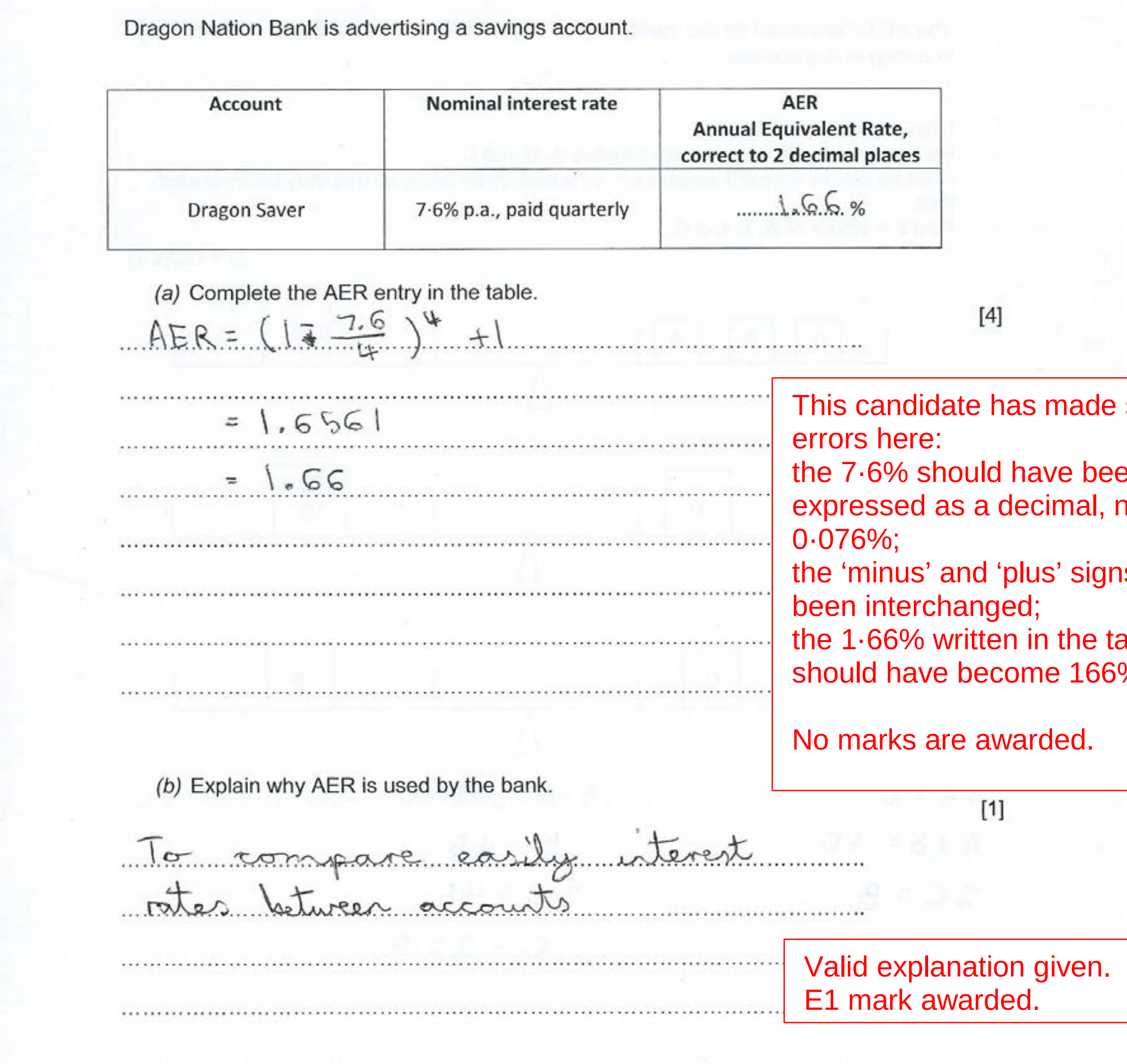
$= 7 \cdot 82\ \%$

(b) Explain why AER is used by the bank. [1]

So people can easily compare interest rates from different banks

Annotated candidate responses

Candidate A



Dragon Nation Bank is advertising a savings account.

Account	Nominal interest rate	AER Annual Equivalent Rate, correct to 2 decimal places
Dragon Saver	7·6% p.a., paid quarterly	1.66 %

(a) Complete the AER entry in the table. [4]

$AER = \left(1 \mp \frac{7.6}{4}\right)^4 + 1$

$= 1.6561$

$= 1.66$

This candidate has made several errors here:
the 7·6% should have been expressed as a decimal, namely 0·076%;
the ‘minus’ and ‘plus’ signs have been interchanged;
the 1·66% written in the table should have become 166%.

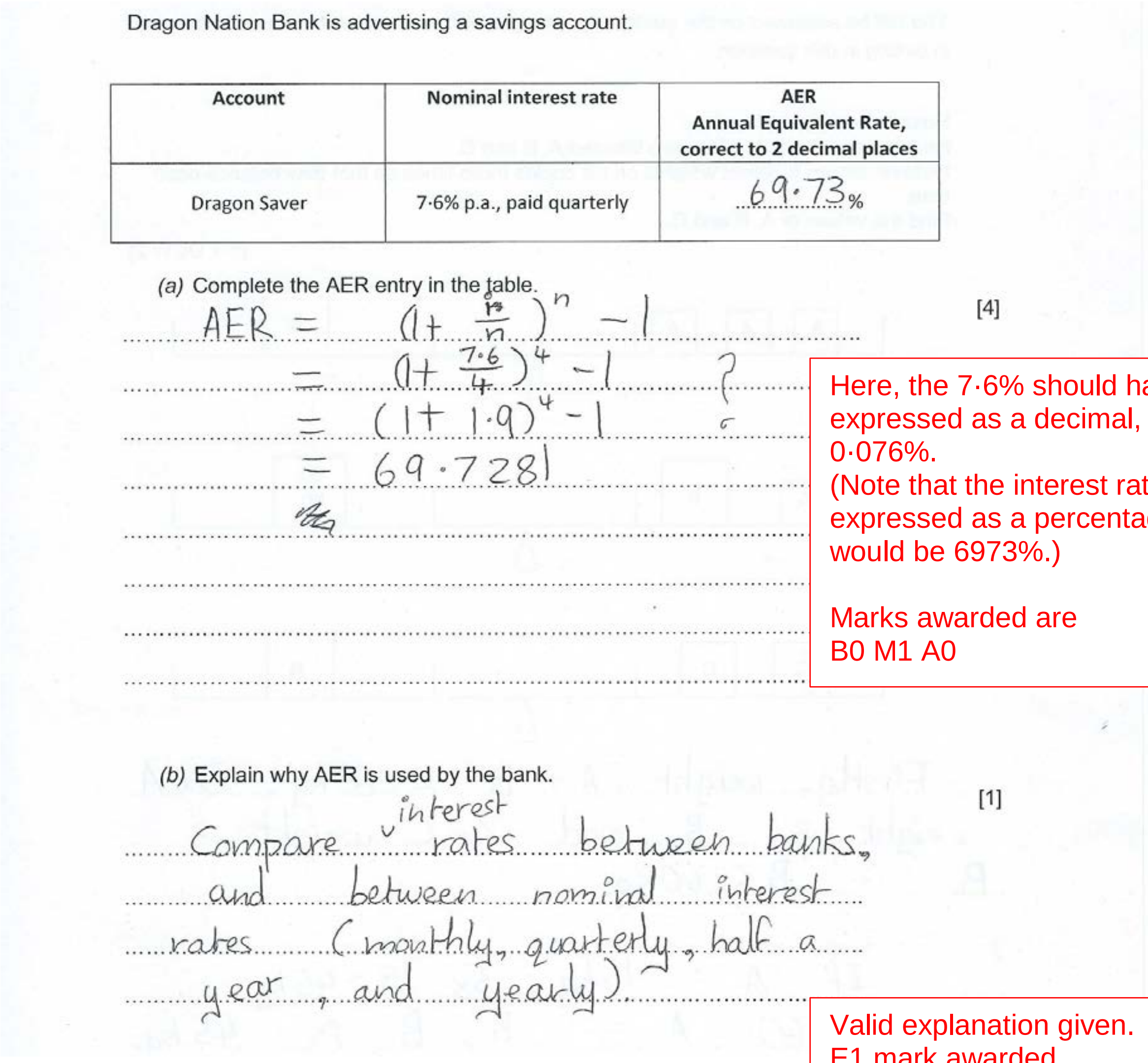
No marks are awarded.

(b) Explain why AER is used by the bank. [1]

To compare easily interest rates between accounts

Valid explanation given.
E1 mark awarded.

Candidate B



Dragon Nation Bank is advertising a savings account.

Account	Nominal interest rate	AER Annual Equivalent Rate, correct to 2 decimal places
Dragon Saver	7·6% p.a., paid quarterly	69·73%

(a) Complete the AER entry in the table. [4]

$AER = (1+ \frac{r}{n})^n - 1$

$= (1+ \frac{7\cdot6}{4})^4 - 1$?

$= (1+ 1\cdot9)^4 - 1$

$= 69\cdot7281$

(b) Explain why AER is used by the bank. [1]

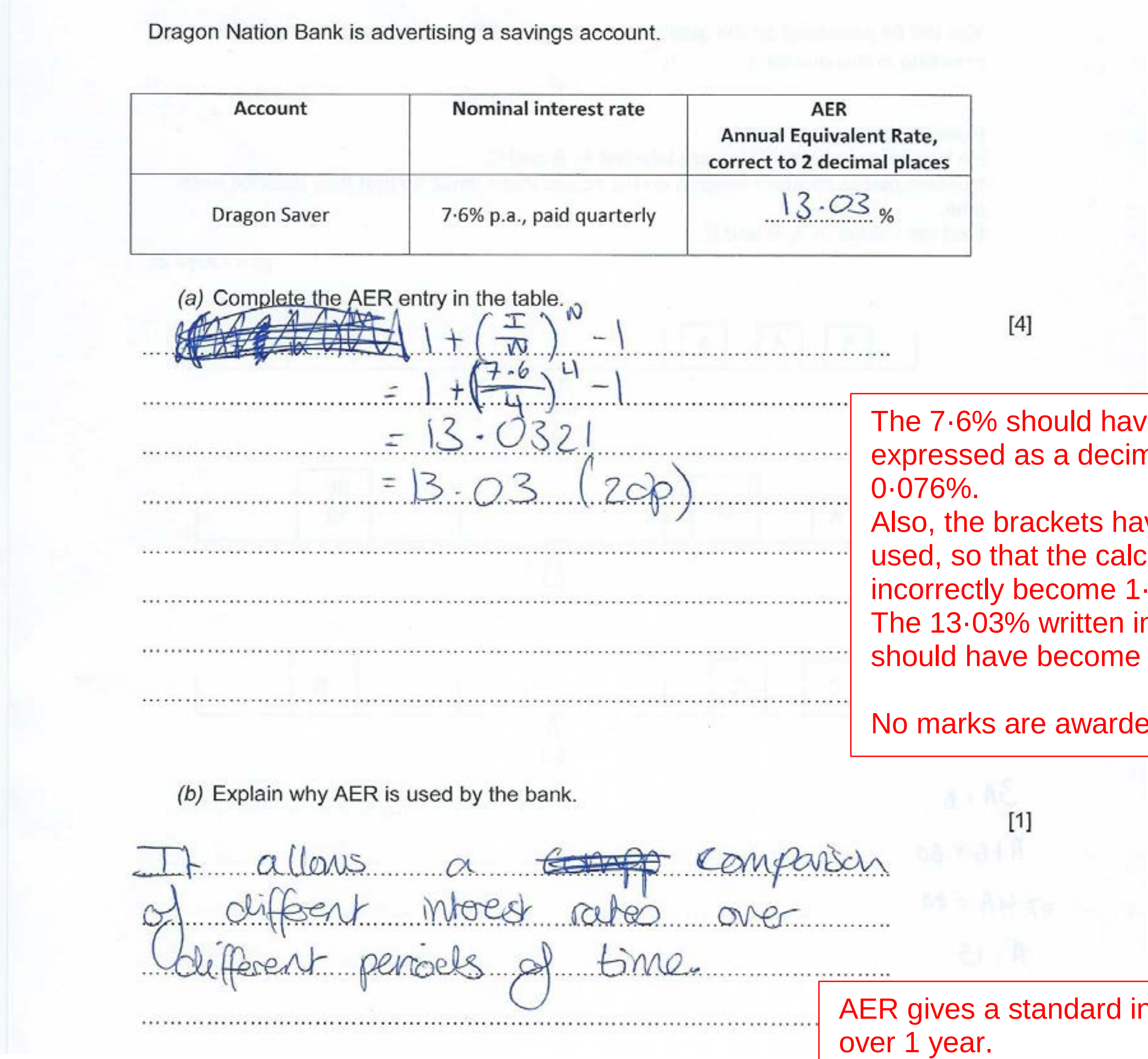
Compare interest rates between banks, and between nominal interest rates (monthly, quarterly, half a year, and yearly).

Here, the 7·6% should have been expressed as a decimal, namely 0·076%.
(Note that the interest rate expressed as a percentage here would be 6973%.)

Marks awarded are
B0 M1 A0

Valid explanation given.
E1 mark awarded.

Candidate C



Dragon Nation Bank is advertising a savings account.

Account	Nominal interest rate	AER Annual Equivalent Rate, correct to 2 decimal places
Dragon Saver	7·6% p.a., paid quarterly	13·03 %

(a) Complete the AER entry in the table. [4]

$1 + \left(\frac{I}{N}\right)^{N} - 1$

$= 1 + \left(\frac{7\cdot6}{4}\right)^{4} - 1$

$= 13\cdot0321$

$= 13\cdot03$ (2dp)

The 7·6% should have been expressed as a decimal, namely 0·076%.
Also, the brackets have been mis-used, so that the calculation has incorrectly become $1\cdot9^{4}$.
The 13·03% written in the table should have become 1303%.

No marks are awarded.

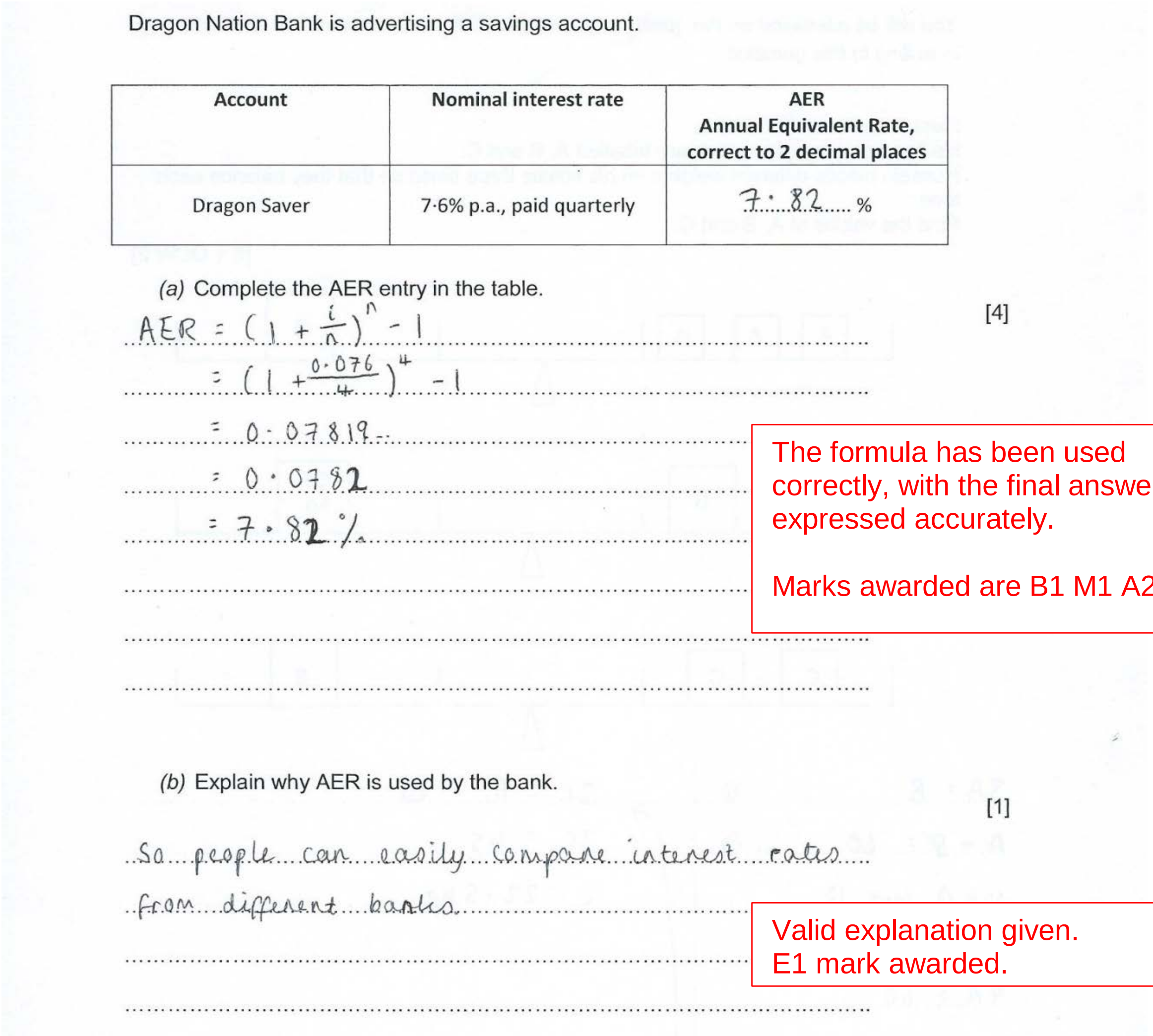
(b) Explain why AER is used by the bank. [1]

It allows a ~~comp~~ comparison of different interest rates over different periods of time.

AER gives a standard interest rate over 1 year.

No mark awarded.

Candidate D



Dragon Nation Bank is advertising a savings account.

Account	Nominal interest rate	AER Annual Equivalent Rate, correct to 2 decimal places
Dragon Saver	7·6% p.a., paid quarterly	7·82 %

(a) Complete the AER entry in the table. [4]

$AER = (1 + \frac{i}{n})^n - 1$

$= (1 + \frac{0\cdot076}{4})^4 - 1$

$= 0\cdot07819...$

$= 0\cdot0782$

$= 7\cdot82\ \%$

The formula has been used correctly, with the final answer expressed accurately.

Marks awarded are B1 M1 A2.

(b) Explain why AER is used by the bank. [1]

So people can easily compare interest rates from different banks.

Valid explanation given.
E1 mark awarded.

3.2 VENN DIAGRAMS

Specification statement (Foundation, Intermediate and Higher tiers, Mathematics - Numeracy and Mathematics) under 'Number':

Understanding and using Venn diagrams to solve problems.

Specification statement (Foundation, Intermediate and Higher tiers, Mathematics only) under 'Statistics':

Use Venn diagrams or other diagrammatic representations of compound events.

Note that any related probability question can only be set on a Mathematics paper (as probability is not specified under Mathematics - Numeracy).

Notes

A Venn diagram provides a means of classifying items of data which may or may not share common properties.

Candidates at all three tiers should be familiar with the terms **universal set** (denoted by Ɛ) and **event** and be able to answer questions involving 2 or 3 sets.

Candidates at the Intermediate and Higher tiers should be familiar with set notation A, B, A', B', $A\cap B$, $A\cup B$, $A'\cap B$, $A\cup B'$ and the terms **union**, **intersection** and **complement**. They should be able to identify these on Venn diagrams involving 2 or 3 sets.

.

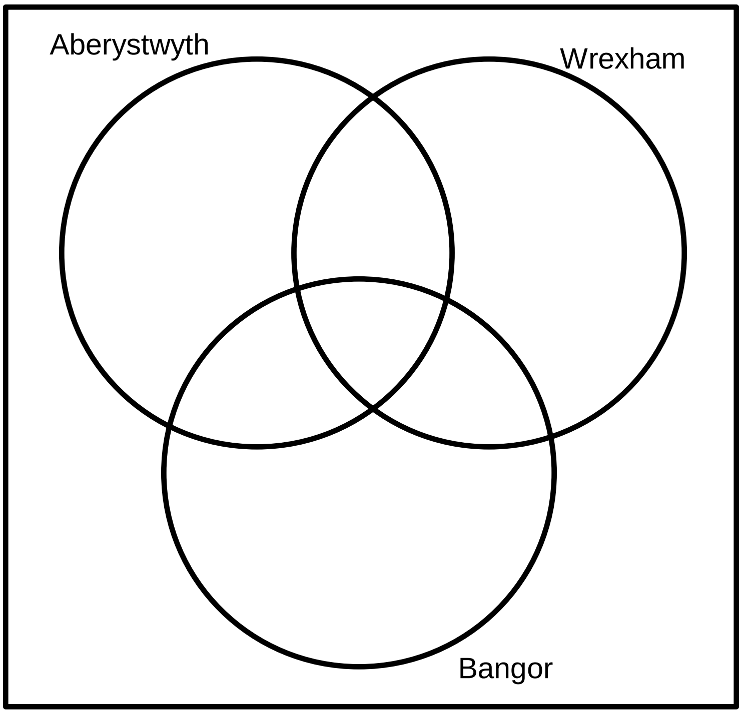
Examples

1. Use a Venn diagram to find the highest common factor (HCF) and lowest common multiple (LCM) of 36 and 48.

2. 30 pupils were asked which town they had visited in the last 2 years: Aberystwyth, Bangor or Wrexham.

   2 pupils had visited all three cities.
   1 pupil had visited Wrexham and Bangor but not Aberystwyth.
   4 pupils had visited Aberystwyth and Wrexham but not Bangor.
   13 pupils had visited Wrexham.
   26 pupils had visited at least one of the cities.
   2 pupils had visited Bangor but not Aberystwyth or Wrexham.
   8 pupils had visited Bangor.

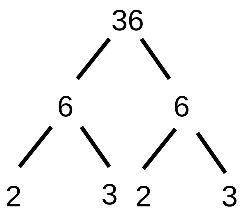
   (a) How many pupils had visited Aberystwyth, but not Wrexham or Bangor?

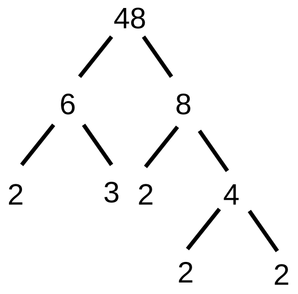


   (b) One pupil was selected at random. Given that the pupil had visited Wrexham, what was the probability that he had also visited Aberystwyth?

Solutions

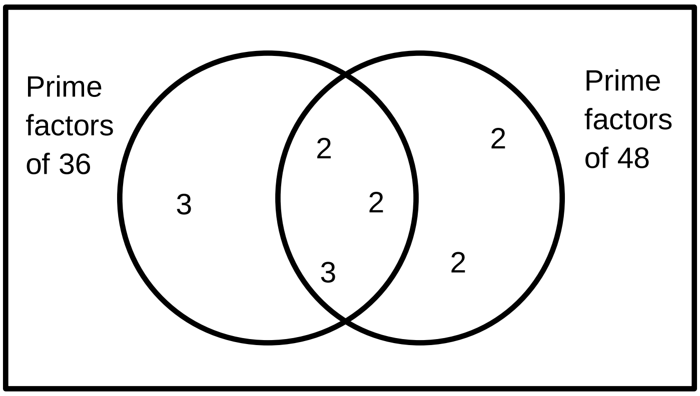
1. First we need to write 36 and 48 as products of primes.





This gives $36 = 2^2 \times 3^2$ and $48 = 2^4 \times 3$

We then construct a Venn diagram, with the sets (circles) labelled as 'Prime factors of 36' and 'Prime factors of 48'.



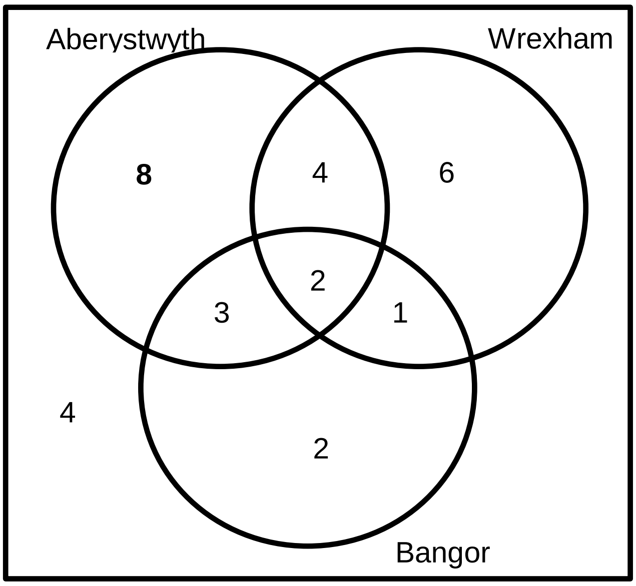
The product of the numbers in the intersection gives the HCF

HCF = 2 × 2 × 3 = 12

The product of all the numbers in the diagram gives the LCM

LCM = 2 × 2 × 2 × 2 × 3 × 3 = 144

2.



(a) Completing the Venn diagram gives us 8 pupils who had visited Aberystwyth, but not Wrexham or Bangor.

(b) 13 pupils had visited Wrexham and, of these, 4 + 2 = 6 had also visited Aberystwyth.

Probability = $\frac{6}{13}$

Examples of examination questions on Venn diagrams

1. *SAMs 1 Mathematics Unit 2 Foundation and Intermediate*

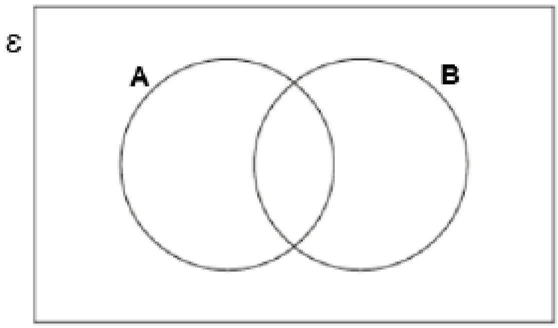
The universal set, Ɛ = {6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,18}

Set A is the multiples of 3.
Set B is the multiples of 4.

(a) Complete the Venn diagram. [4]

....

....



(b) What is the probability that a number selected at random from this universal set is a multiple of 3 but not a multiple of 4? [2]

....

....

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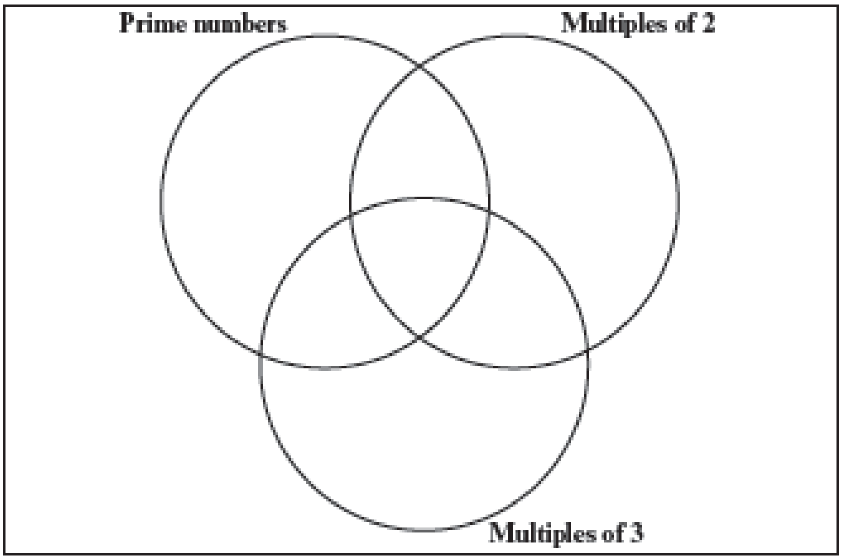
2. *January 2012 Methods in Mathematics Unit 1 Higher*

(a) (i) Place each of the whole numbers 42, 43, 44, 45, 46, 47, 48, 49, 50 in the correct positions in the Venn diagram.



[3]

(ii) A whole number is selected at random from the set {42, 43, 44, 45, 46, 47, 48, 49, 50}.

Find the probability that the number selected is

a prime number, ...

not a prime number, ...

a prime number that is also a multiple of 3. ....

[3]

(b) A die has previously been used and shown to be fair.
This fair die is thrown a further 60 times; a six is scored on the die on 15 of these throws.
Giving a reason for your answer, write down the probability that a six is scored on the next throw.

....

....

....

....

[2]

3. *January 2014 Methods in Mathematics Unit 1 Higher*

The universal set, ε = {22, 23, 24, 25, 26, 27, 28, 29, 30}.

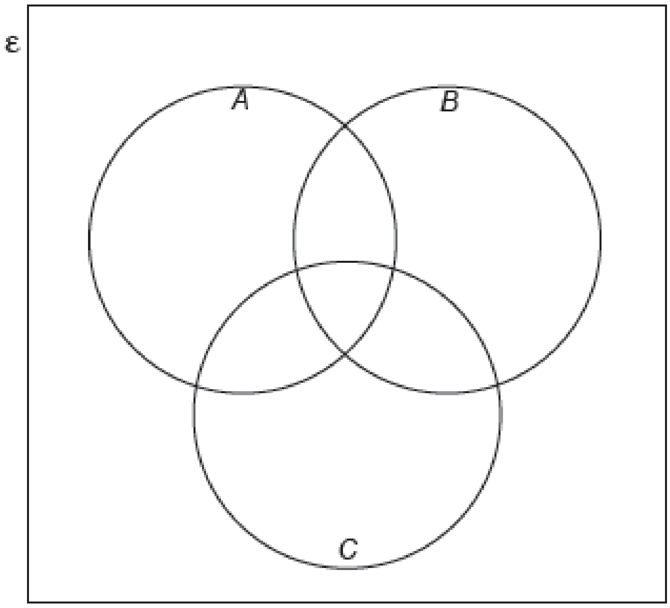
Within this universal set ε,

- set *A* is the multiples of 2
- set *B* is the multiples of 4
- set *C* is the multiples of 5

(a) Complete the Venn diagram. [3]

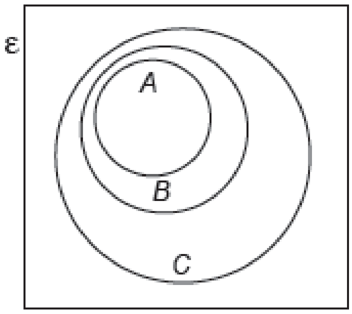
……

……

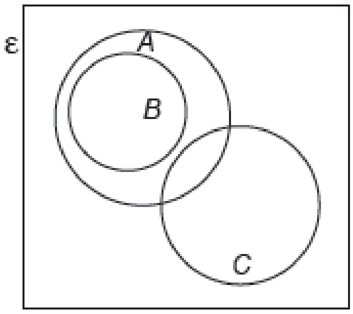


(b) Which one of the following Venn diagrams could also be used to represent the sets ε, *A*, *B* and *C*?
You must give a reason for your choice. [2]

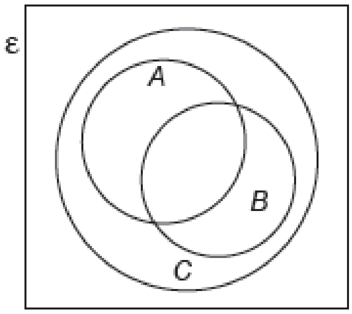
Venn Diagram 1



Venn Diagram 2



Venn Diagram 3



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(c) A whole number is selected at random from the universal set
ε = {22, 23, 24, 25, 26, 27, 28, 29, 30}.

Find the probability that the number selected is: [3]

a multiple of 2 but not a multiple of 4 ……………………

not a multiple of 5 ……………………

a multiple of 5 and a multiple of 2 ……………………

4. *January 2015 Applications of Mathematics Unit 1 Higher*

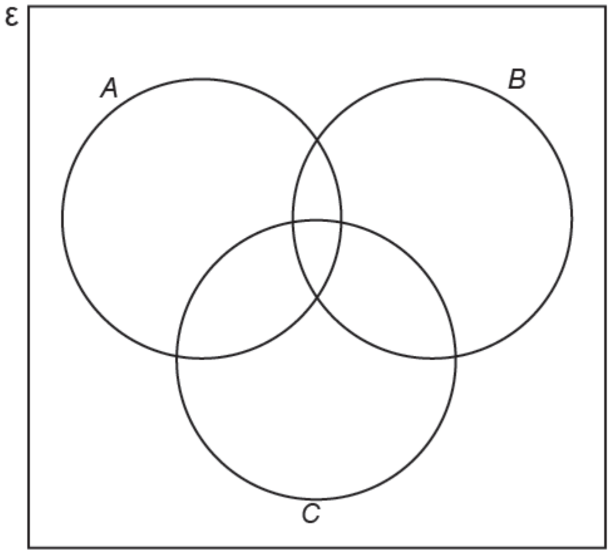
Berlin's main railway station is known as the Hauptbahnhof.
Bellevue and Wildau are two railway stations in opposite directions from the Hauptbahnhof.

On a particular day,
- trains leave the Hauptbahnhof to Bellevue every 14 minutes
- trains leave the Hauptbahnhof to Wildau every 12 minutes.

A train to Bellevue and a train to Wildau both leave the Hauptbahnhof at 10:00.

When will a train to Bellevue and a train to Wildau next leave the Hauptbahnhof at the same time? [4]

5. *January 2015 Methods in Mathematics Unit 1 Higher*



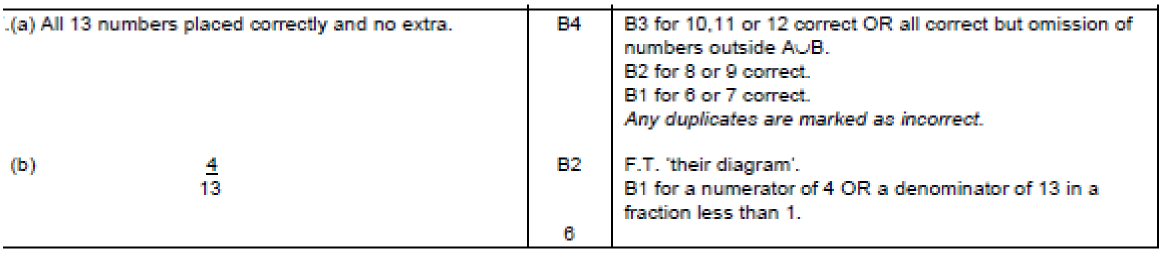
An outline of a Venn diagram is shown above.
You are given the following information.

- $P(A \cup B \cup C)' = 0{\cdot}01$
- $P(A \cap B \cap C) = 0{\cdot}2$
- $P(B \cap C) = 0{\cdot}5$
- $P(A \cap B) = 0{\cdot}3$
- $P(A \cup C) = 0{\cdot}65$

Calculate P(*B*). [7]

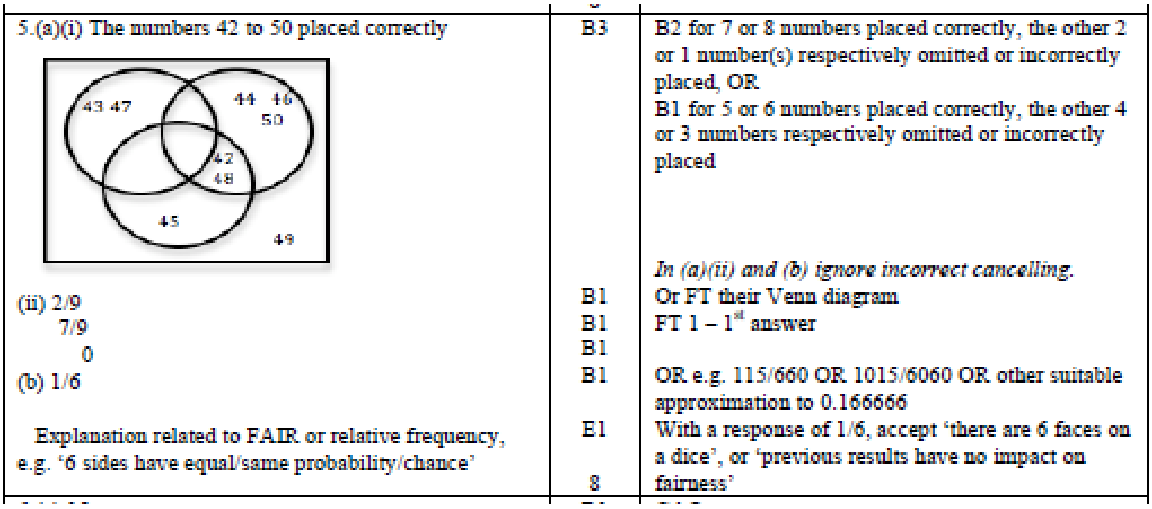
Mark schemes for examination questions on Venn diagrams

1. *SAMs 1 Mathematics Unit 1 Higher*



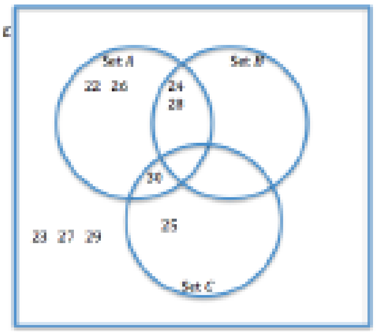
.(a) All 13 numbers placed correctly and no extra.	B4	B3 for 10,11 or 12 correct OR all correct but omission of numbers outside A∪B. B2 for 8 or 9 correct. B1 for 6 or 7 correct. *Any duplicates are marked as incorrect.*
(b) $\frac{4}{13}$	B2	F.T. 'their diagram'. B1 for a numerator of 4 OR a denominator of 13 in a fraction less than 1.
	6	

2. *January 2012 Methods in Mathematics Unit 1 Higher*



5.(a)(i) The numbers 42 to 50 placed correctly	B3	B2 for 7 or 8 numbers placed correctly, the other 2 or 1 number(s) respectively omitted or incorrectly placed, OR B1 for 5 or 6 numbers placed correctly, the other 4 or 3 numbers respectively omitted or incorrectly placed
		In (a)(ii) and (b) ignore incorrect cancelling.
(ii) 2/9	B1	Or FT their Venn diagram
7/9	B1	FT 1 – 1st answer
0	B1	
(b) 1/6	B1	OR e.g. 115/660 OR 1015/6060 OR other suitable approximation to 0.166666
Explanation related to FAIR or relative frequency, e.g. '6 sides have equal/same probability/chance'	E1	With a response of 1/6, accept 'there are 6 faces on a dice', or 'previous results have no impact on fairness'
	8	

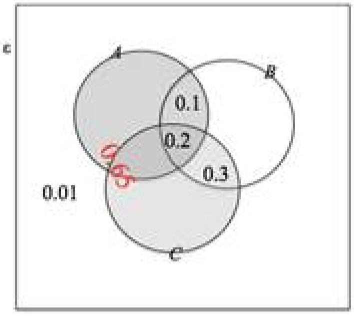
3. *January 2014 Methods in Mathematics Unit 1 Higher*

5(a) All 9 numbers placed correctly 	B3	B2 for any 7 or 8 numbers placed correctly, the other numbers omitted or incorrectly placed, OR B1 for any 5 or 6 numbers placed correctly, the other numbers omitted or incorrectly placed. *Any ambiguous duplicates are marked as an incorrect placement for that number*
(b) Venn diagram 2 AND full reason, e.g. 'multiples of 4 are a subset of multiples of 2 and there is a multiple of 2 which is a multiple of 5', or 'set B is a subset of set A, and set A intersects with set C', or 'A & B share some of the numbers, but C only shares numbers with A', or 'C & B have nothing in common, and B shares everything with A'	E2	OR selects Venn diagram 2 and explains why the other 2 Venn diagrams are not selected E1 for choice of Venn diagram 2 AND a partial reason, i.e. only mentions 1 aspect or attempts an explanation e.g. '4 times table is within 2 times table', or 'shows which of A are within 4 times table', or '22 is in A but not in C', or 'no multiples of 4 in C' OR E1 for selection of Venn diagram 2 and explains why 1 of the other 2 Venn diagrams are not selected *Accept informal words such as 'within' for 'subset', 'overlap' for 'intersection'*
(c) 3/9 (=1/3)	B1	FT their Venn diagram.
7/9	B1	FT a slip in the denominator used consistently
1/9	B1	FT a slip in the denominator used consistently
	8	

4. *January 2015 Applications of Mathematics Unit 1 Higher*

Considering multiples of 12 and 14, e.g. sight of 12, 24, 36, .. AND 14, 28, 42, .., OR Looking at factors of 12 and 14, e.g. sight of 2×6 AND 2×7	S1	At least 3 correct multiples for both
Correct list of multiples of 12 to at least 72, or multiple 72 AND Correct list of multiples of 14 to at least 70, or multiple 70 , OR Sight of 2×6×7	M1	12, 24, 36, 48, 60, 72, 84 14, 28, 42, 56, 70, 84 *Alternative method: Use Venn diagram to place prime factors of 12 and 14 correctly.*
Sight of 84 (as common multiple or number of minutes)	A1	OR 1 hour 24 minutes
Time 11:24	A1	FT time from 10:00 for their number of minutes provided S1 and M1 awarded *If no marks SC2 for an answer of 12(:)48, OR SC1 for sight of 2hours 48minutes* *No marks for sight of 168(minutes) alone.*

5. *January 2015 Methods in Mathematics Unit 1 Higher*

17.  Note: Shaded $P(A\cup C) = 0.65$	B1 B3	Evidence for B marks may be seen in working Correct indication of 0.01 For Venn diagram shown, correct indication of • 0.1, 0.2 and 0.3, or • the 0.3 shown and $A\cap B$ is 0.3 used • the 0.1 shown and $B\cap C$ as 0.5 used <u>If not B3 then mark individually as follows</u>: B1 for correct indication of 0.2, B1 for correct indication of 0.3, B1 for correct indication of 0.1
Method to find B not intersecting with A nor C, e.g. 1 – 0.65 - 0.01 (= 0.34) $P(B) = 0.34 + 0.1 + 0.2 + 0.3$ $P(B) = 0.94$	M1 M1 A1	*Allow "P(B)" = 0.34 (not from 2 – 1.66)* (FT 'their 0.34') CAO *Alternative* $P((A\cup C)\cap B') = 0.65 - 0.2 - 0.3 - 0.1$ (= 0.05) M1 $P(B) = 1 - 0.01 - 0.05 = 0.94$ (FT 'their 0.05') M1 $P(B) = 0.94$ (CAO) A1
	7	

Further examples of questions can be found on the WJEC website in Unit 1 Methods in Mathematics papers (4363/01 and 4363/02) from January 2011 onwards (January and June series).

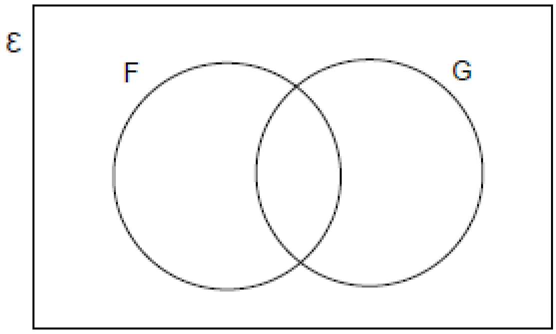
Worked and marked example on Venn diagrams

SAMs 1 Mathematics Unit 2 Higher

30 students in a Year 11 class have decided which subjects they are going to study next year.

- 21 have decided to study French (F)
- 12 have decided to study German (G)
- 5 have decided not to study either French or German.

(a) Complete the Venn diagram below to show this information.
The universal set Ɛ contains all the students in the class. [2]



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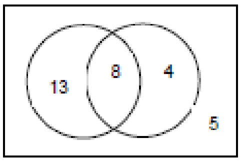
(b) Given that a student, chosen at random, has decided to study French, what is the probability that this student has also decided to study German? [2]

....

....

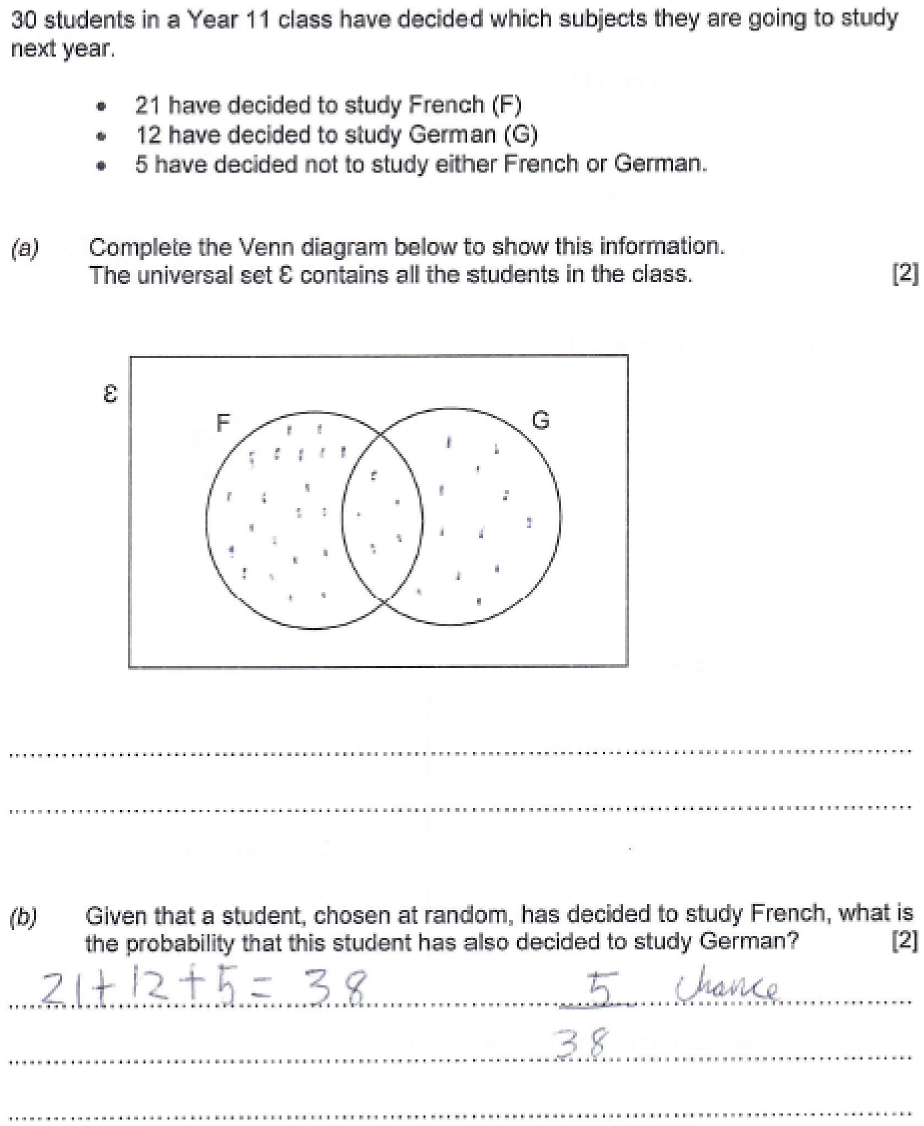
....

Mark scheme

14(a) 	B2	For all correct. B1 for two or three correct.
(b) 8/21	B2	F.T. their complete Venn diagram. B1 for a numerator of 8 in a fraction < 1. B1 for a denominator of 21 in a fraction < 1.
	4	

Candidate responses

Candidate A



30 students in a Year 11 class have decided which subjects they are going to study next year.

- 21 have decided to study French (F)
- 12 have decided to study German (G)
- 5 have decided not to study either French or German.

(a) Complete the Venn diagram below to show this information.
The universal set ε contains all the students in the class. [2]

……………………………………………………………………………………………

……………………………………………………………………………………………

(b) Given that a student, chosen at random, has decided to study French, what is the probability that this student has also decided to study German? [2]

21+12+5 = 38 $\frac{5}{38}$ chance

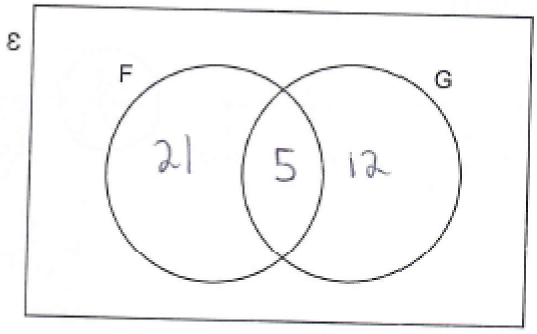
……………………………………………………………………………………………

Candidate B

14. 30 students in a Year 11 class have decided which subjects they are going to study next year.

- 21 have decided to study French (F)
- 12 have decided to study German (G)
- 5 have decided not to study either French or German.

(a) Complete the Venn diagram below to show this information.
The universal set Ɛ contains all the students in the class. [2]



(b) Given that a student, chosen at random, has decided to study French, what is the probability that this student has also decided to study German? [2]

30 Student

21 french. 57 %. 0.57.

12 German.

21 ÷ 30 = 0.7 french.

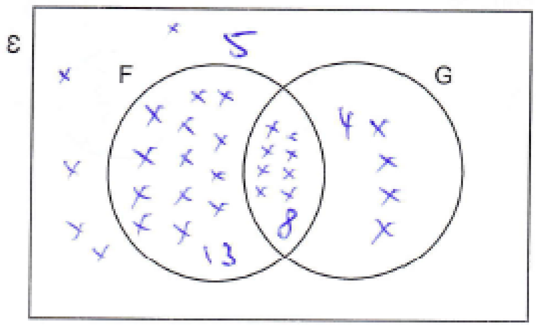
12 ÷ 21. 0.57

Candidate C

30 students in a Year 11 class have decided which subjects they are going to study next year.

- 21 have decided to study French (F)
- 12 have decided to study German (G)
- 5 have decided not to study either French or German.

(a) Complete the Venn diagram below to show this information.
The universal set ℰ contains all the students in the class. [2]



21+ 12 + 5 = 38

21 − 8 = 13   12 − 8 = 4

(b) Given that a student, chosen at random, has decided to study French, what is the probability that this student has also decided to study German? [2]

$\frac{8}{21}$ / 38 %.

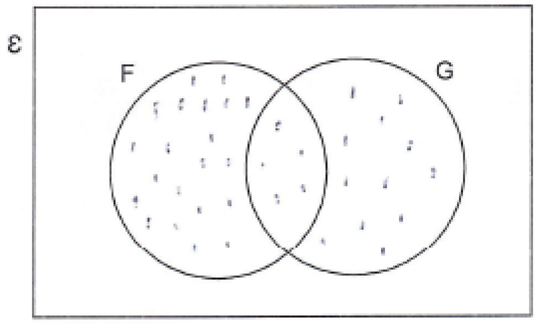
Annotated candidate responses

Candidate A

30 students in a Year 11 class have decided which subjects they are going to study next year.

- 21 have decided to study French (F)
- 12 have decided to study German (G)
- 5 have decided not to study either French or German.

(a) Complete the Venn diagram below to show this information. The universal set ℰ contains all the students in the class. [2]



This candidate appears to have chosen to place dots in the circles, without evaluating the number of dots in each case. Even though this could be an effective method, it is not a valid way of presenting a final answer. (Furthermore, the numbers of dots do not seem to be correct e.g. 5 in the intersection.)

No marks are awarded here.

(b) Given that a student, chosen at random, has decided to study French, what is the probability that this student has also decided to study German? [2]

21+12+5= 38 $\frac{5}{38}$ chance

Following through from the candidate's Venn diagram, the numerator of 5 is correct, but not the denominator.

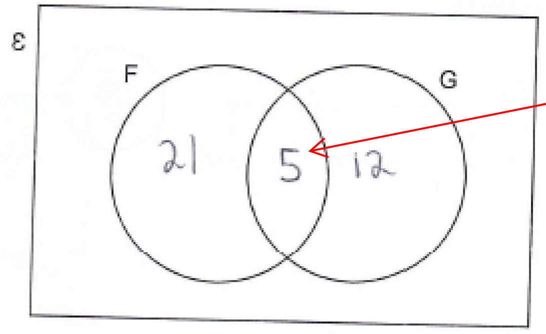
B1 is awarded.

Candidate B

14. 30 students in a Year 11 class have decided which subjects they are going to study next year.

- 21 have decided to study French (F)
- 12 have decided to study German (G)
- 5 have decided not to study either French or German.

(a) Complete the Venn diagram below to show this information. The universal set ℰ contains all the students in the class. [2]



The candidate has not understood that the ‘5’ belongs outside the circles. There are no correct entries.

No marks are awarded here.

(b) Given that a student, chosen at random, has decided to study French, what is the probability that this student has also decided to study German? [2]

30 Student

21 french. 57 % 0.57.

12 German.

21 ÷ 30 = 0.7 french.

12 ÷ 21. 0.57

The answer of 12 / 21 does not follow from the candidate’s Venn diagram.

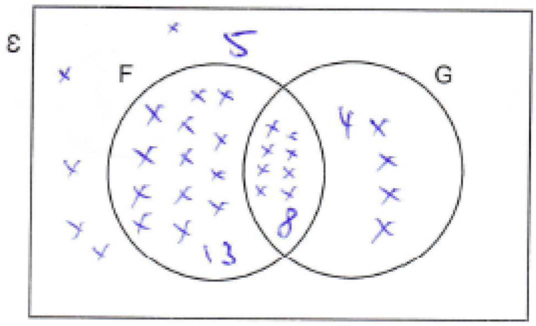
No marks are awarded here.

Candidate C

30 students in a Year 11 class have decided which subjects they are going to study next year.

- 21 have decided to study French (F)
- 12 have decided to study German (G)
- 5 have decided not to study either French or German.

(a) Complete the Venn diagram below to show this information.
The universal set ℰ contains all the students in the class. [2]



This candidate appears to have chosen to place crosses in the circles, and has then evaluated the number of crosses in each case. All the numbers entered are correct.

Both marks are awarded here.

21+ 12 + 5 = 38

21 − 8 = 13 12 − 8 = 4

(b) Given that a student, chosen at random, has decided to study French, what is the probability that this student has also decided to study German? [2]

$\frac{8}{21}$ / 38 %.

Both the numerator and denominator are correct.

Both marks are awarded here.

3.3 EQUATIONS OF PERPENDICULAR LINES

Specification statement. (Intermediate and Higher tiers, Mathematics only)

Identifying the equation of lines parallel or perpendicular to a given line, to satisfy given conditions

Notes

The gradient of a straight line is a measure of its steepness.

Fact: the gradients of perpendicular lines have a product of –1.

The gradient is the coefficient of x in the equation $y = mx + c$
(The *coefficient* of x is the number that multiplies it.)

So, for 2 different equations of straight lines,

$y = m_1x + c_1$ and $y = m_2x + c_2$,

the lines are perpendicular if $\boxed{m_1m_2 = -1}$

Example

For each of the following pairs of equations, decide whether or not they represent perpendicular lines.

1. $y = 2x + 3$ and $y = -\frac{1}{2}x + 7$

2. $y = -10x + 3$ and $y = 0\cdot1x - 1$

3. $y + x = 2$ and $y = -x + 1$

4. $y = \frac{3}{4}x + 3$ and $y = -\frac{4}{3}x + 7$

5. $2y = -x + 4$ and $y = 2x - 9$

6. $4y + 3x + 8 = 0$ and $3y - 4x - 6 = 0$

7. $x - 32y = 0$ and $y - 16x = 32$

8. $5y - 8x = 0$ and $y = \frac{1}{8}x - 5$

Solutions

1. The gradients are 2 and $-\frac{1}{2}$, with a product of $2 \times -\frac{1}{2} = -1$.

Answer: perpendicular

*** Notice that it does not matter that the y-intercepts (the values of c_1 and c_2) are different.***

2. The gradients are -10 and $0{\cdot}1$, with a product of $-10 \times 0{\cdot}1 = -1$.

Answer: perpendicular

3. This time, the first equation needs to be rearranged into the form $y = mx + c$, so that the gradient is easy to identify.

$y + x = 2$ becomes $y = -x + 2$.

The gradients are -1 and -1, with a product of $-1 \times -1 = 1$.

Answer: NOT perpendicular

(In fact, the gradients are equal, therefore these lines are PARALLEL).

4. The gradients are $\frac{3}{4}$ and $-\frac{4}{3}$, with a product of $\frac{3}{4} \times -\frac{4}{3} = -1$.

Answer: perpendicular

5. $2y = -x + 4$ becomes $y = -\frac{1}{2}x + 2$.

The gradients are $-\frac{1}{2}$ and 2, with a product of $-\frac{1}{2} \times 2 = -1$.

Answer: perpendicular

6. This time, both equations need to be rearranged into the form $y = mx + c$.

$4y + 3x + 8 = 0$ becomes $y = -\frac{3}{4}x - 2$

$3y - 4x - 6 = 0$ becomes $y = \frac{4}{3}x + 2$

The gradients are $-\frac{3}{4}$ and $\frac{4}{3}$, with a product of $-\frac{3}{4} \times \frac{4}{3} = -1$.

Answer: perpendicular

7. $x - 32y = 0$ becomes $y = \frac{1}{32}x$

$y - 16x = 32$ becomes $y = 16x + 32$

The gradients are $\frac{1}{32}$ and 16, with a product of $\frac{1}{32} \times 16 = \frac{1}{2}$.

Answer: NOT perpendicular

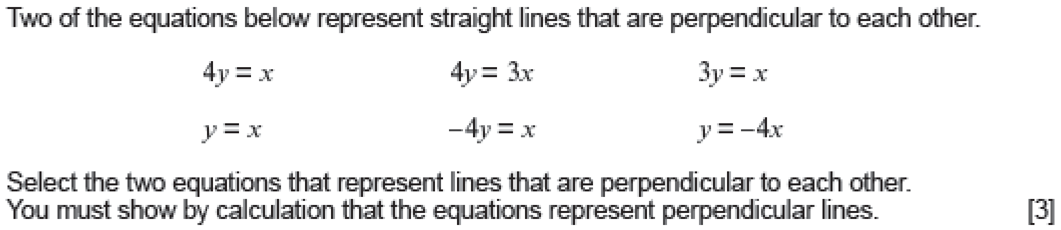
8. $5y - 8x = 0$ becomes $y = \frac{8}{5}x$

The gradients are $\frac{8}{5}$ and $\frac{1}{8}$, with a product of $\frac{8}{5} \times \frac{1}{8} = \frac{1}{5}$.

Answer: NOT perpendicular

Examples of examination questions on perpendicular lines

1. *January 2014 Methods in Mathematics Unit 1 Higher*



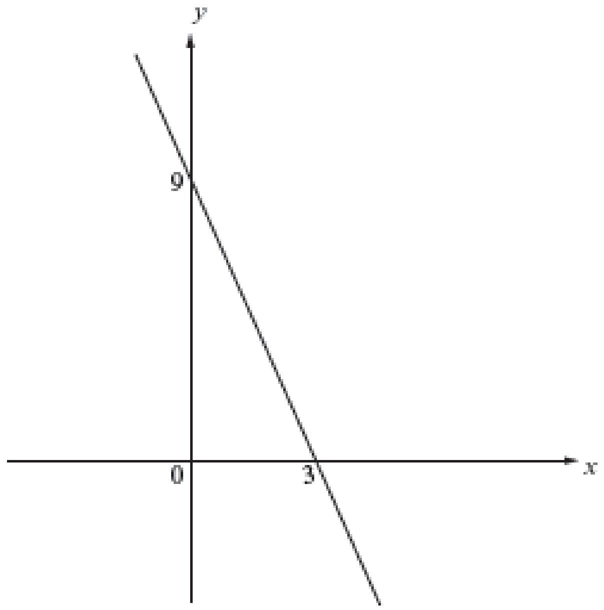
Two of the equations below represent straight lines that are perpendicular to each other.

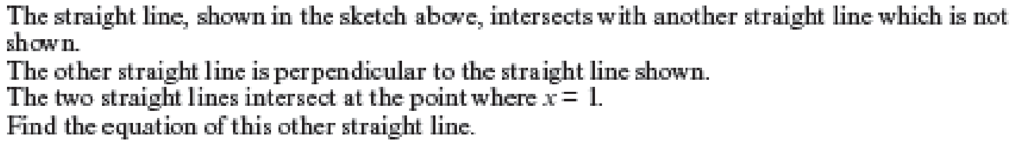
$4y = x$ $\quad$ $4y = 3x$ $\quad$ $3y = x$

$y = x$ $\quad$ $-4y = x$ $\quad$ $y = -4x$

Select the two equations that represent lines that are perpendicular to each other.
You must show by calculation that the equations represent perpendicular lines. [3]

2. *January 2012 Methods in Mathematics Unit 1 Higher*

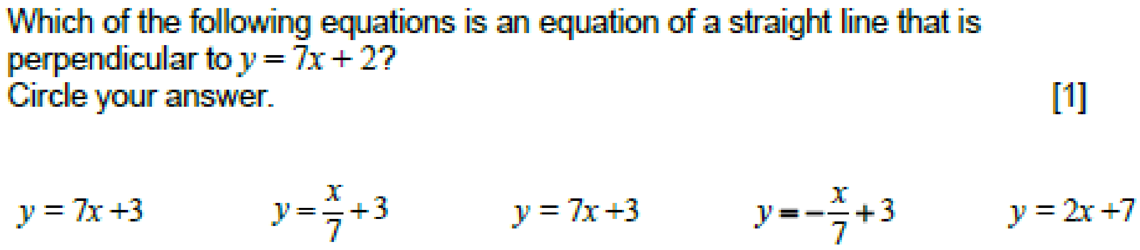




The straight line, shown in the sketch above, intersects with another straight line which is not shown.
The other straight line is perpendicular to the straight line shown.
The two straight lines intersect at the point where $x = 1$.
Find the equation of this other straight line.

[8]

3. *SAMs 1 Mathematics Unit 2 Intermediate and Higher*

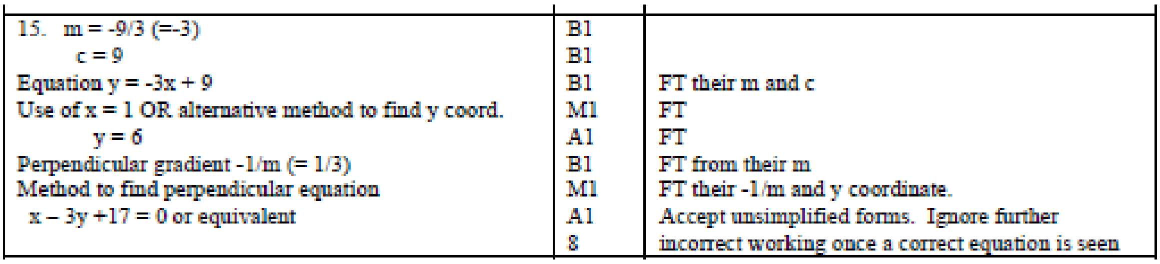


Which of the following equations is an equation of a straight line that is perpendicular to $y = 7x + 2$?
Circle your answer. [1]

$y = 7x + 3$ $\quad$ $y = \frac{x}{7} + 3$ $\quad$ $y = 7x + 3$ $\quad$ $y = -\frac{x}{7} + 3$ $\quad$ $y = 2x + 7$

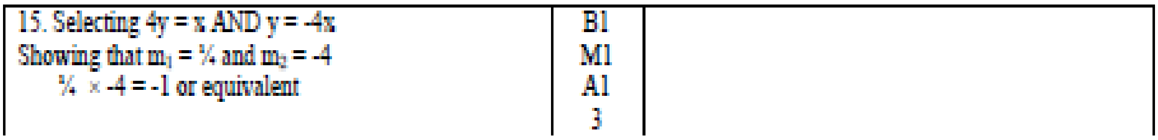
Mark schemes for examination questions on perpendicular lines

1. *January 2012 Methods in Mathematics Unit 1 Higher*



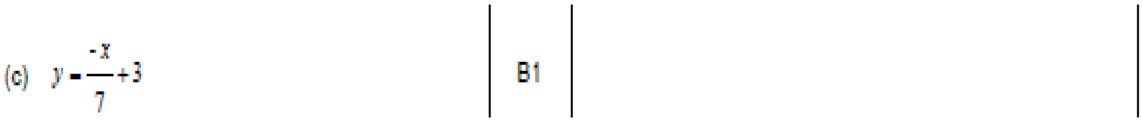
15. m = -9/3 (=-3)	B1	
c = 9	B1	
Equation y = -3x + 9	B1	FT their m and c
Use of x = 1 OR alternative method to find y coord.	M1	FT
y = 6	A1	FT
Perpendicular gradient -1/m (= 1/3)	B1	FT from their m
Method to find perpendicular equation	M1	FT their -1/m and y coordinate.
x – 3y +17 = 0 or equivalent	A1	Accept unsimplified forms. Ignore further
	8	incorrect working once a correct equation is seen

2. *January 2014 Methods in Mathematics Unit 1 Higher*



15. Selecting 4y = x AND y = -4x	B1	
Showing that m_1 = ¼ and m_2 = -4	M1	
¼ × -4 = -1 or equivalent	A1	
	3	

3. *SAMs 1 Mathematics Unit 2 Intermediate and Higher*



(c) $y = \frac{-x}{7} + 3$	B1	

Further examples of questions can be found on the WJEC website in Unit 1 Higher Methods in Mathematics papers (4363/02) from January 2011 onwards (January and June series).

3.4 DIMENSIONS

Specification statement (Intermediate and Higher tiers, Mathematics - Numeracy and Mathematics)

Distinguishing between formulae for length, area and volume by considering dimensions.

Example

The letters a, b and c represent lengths.
For each of the following expressions, decide whether it represents a length, area, volume of none of these.

(i) $3ab$

(ii) $\pi c^2 a - b^3$

(iii) $5b^3 + 2ac$

(iv) $4a(b + 2c)$

(v) $\dfrac{b^2 + c^2}{2a}$

(vi) $3c + a - 2b$.

Solution

(i) 3 is a constant and is therefore 'dimensionless' and can be disregarded. The expression becomes 'length × length' (or L^2).
Answer: area

(ii) This time, π is a constant and is therefore 'dimensionless' and can be disregarded. Each term is then 'length × length × length' (or L^3). As the two terms have the same dimension, the expression is 'dimensionally consistent'.
Answer: volume

(iii) Disregarding the 5 and 2 leaves us with 'length × length × length' (or L^3) for the first term, and 'length × length' (or L^2) for the second term. As the two terms have different dimensions, the expression is not dimensionally consistent.
Answer: none of these

(iv) Expanding the brackets gives $4ab + 8ac$. Disregarding the 4 and 8 leaves us with 'length × length' (or L^2) for both terms, making the expression dimensionally consistent.
Answer: area

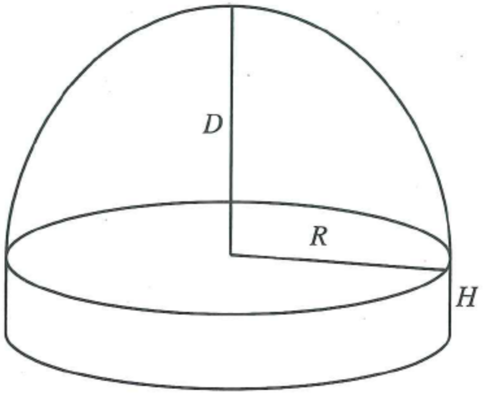
(v) Both terms in the numerator give us 'length × length' (or L^2). Disregarding the 2, the denominator is a 'length' (or L). Dividing then gives 'length' (or L).
Answer: length

(vi) Disregarding the constants, each term is a 'length'.
Answer: length.

Examples of examination questions on dimensions

1. *June 2000 Linear Intermediate*

The diagram shows a solid. The lengths D, R and H are as shown.



One of the following formulae may be used to estimate V, the volume of the solid.

$$V = 3H + 2R + 5D$$
$$V = 3R + 5DR$$
$$V = 3R^2H + 2R^2D$$
$$V = 3R(4D + 5H)$$

(a) Explain why the formula $V = 3H + 2R + 5D$ cannot be used to estimate the volume of the solid.

[1]

(b) State, with a reason, which of the above formulae may be used to estimate the volume of the solid.

[2]

2. *November 2008 Paper 1 Higher (3 tier)*

Each of the following quantities has a particular number of dimensions. Give the number of dimensions of each quantity. The first one has been done for you.

Quantity	Number of dimensions
The capacity of a jug	3
The circumference of a circle	
The volume of a cuboid	
The distance between Cardiff and Builth Wells	
The area of a rectangle	

[2]

3. *June 2005 Paper 1 Higher*

In each of the following formulae, every letter stands for the measurement of a length. By considering the dimensions implied by the formulae, write down, for each case, whether the formula could be for a length, an area, a volume or none of these.
The first one has been done for you.

	Formula could be for
$d^2 + hr$	area
$4d + 3r + 5h$	....
$6rh + 5h^2r$	....
$(d^2 + dh)r$	....
$5rh + 4r^2 - 7rd$	....

[2]

4. *June 1998 Linear Intermediate*

A factory uses wire to make frames for plant covers as shown in the diagram. Each frame has width *W*, depth *D* and uprights of height *H*.

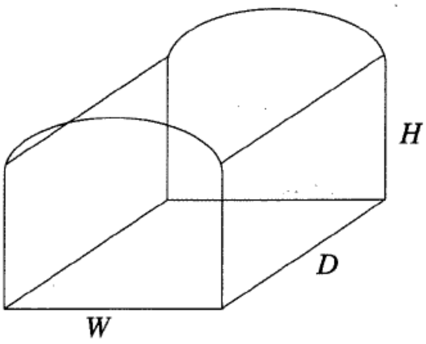


Diagram not drawn to scale

One of these formulae may be used to estimate *L*, the total length of wire required for each frame.

$L = 5W + 4D + 4H$
$L = 5W + 4DH$
$L = 5W(4D + 4H)$
$L = 5WDH$

(a) Explain why the formula $L = 5WDH$ cannot be used to estimate the total length of wire required.

[1]

(b) State, with a reason, which of the above formulae may be used to estimate the total length of wire required.

[2]

5. *June 2008 Paper 1 Higher*

In each of the following formulae, every letter stands for the measurement of a length. By considering the dimensions implied by the formulae, write down, for each case, whether the formulae could be for a length, an area, a volume or none of these.
The first one has been done for you.

	Formula could be for
$r^2 + dh$	area
$r^2\,(2d - h)$	…………………………………………
$3d + 2h - r$	…………………………………………
$5r + 6dr + 2d$	…………………………………………
$6rd - 2d^2 + hr$	…………………………………………

[2]

6. *June 2007 Paper 1 Higher*

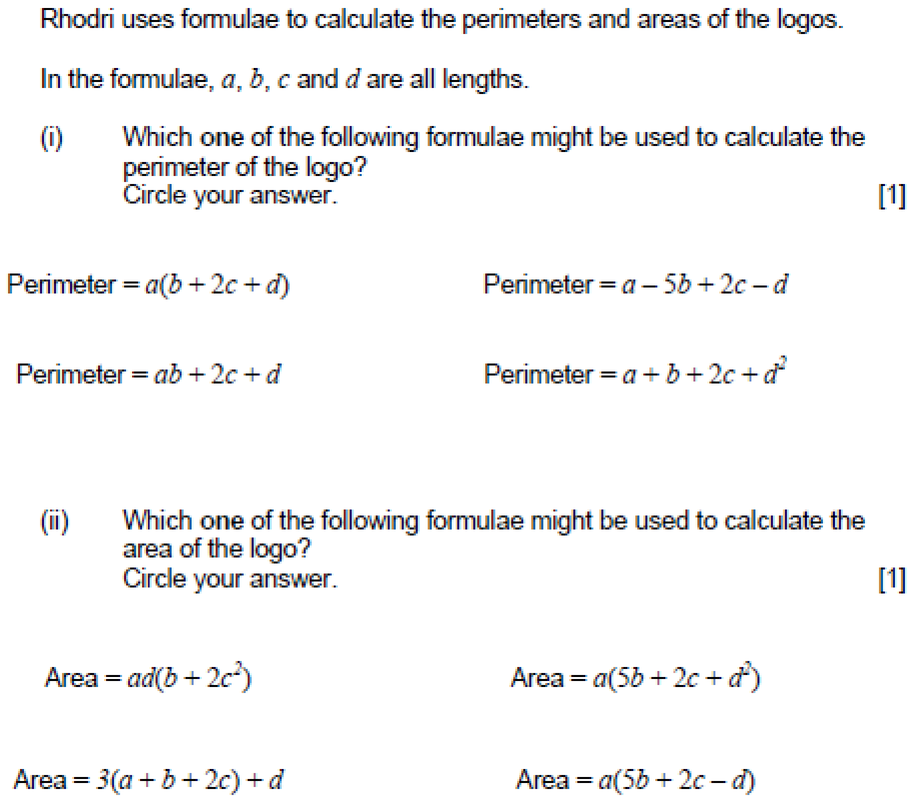
Each of the following quantities has a particular number of dimensions. Give the number of dimensions of each quantity. The first one has been done for you.

Quantity	Number of dimensions
The capacity of a jug.	3
The distance between Cardiff and Mold.	
The area of a field.	
The perimeter of a hexagon.	
The volume of a cuboid.	

[2]

7. *SAMs 1 Mathematics – Numeracy Unit 1 Higher*

A company uses its logo in every part of its business.



Rhodri uses formulae to calculate the perimeters and areas of the logos.

In the formulae, a, b, c and d are all lengths.

(i) Which one of the following formulae might be used to calculate the perimeter of the logo?
Circle your answer. [1]

Perimeter = $a(b + 2c + d)$	Perimeter = $a - 5b + 2c - d$
Perimeter = $ab + 2c + d$	Perimeter = $a + b + 2c + d^2$

(ii) Which one of the following formulae might be used to calculate the area of the logo?
Circle your answer. [1]

Area = $ad(b + 2c^2)$	Area = $a(5b + 2c + d^2)$
Area = $3(a + b + 2c) + d$	Area = $a(5b + 2c - d)$

Mark schemes for examination questions on dimensions

1. *June 2000 Linear Intermediate*

(a) Explanation that the expression (on the right) is for length OR is one-dimensional	E1	
(b) $V = 3R^2H + 2R^2D$	B1	
(Disregarding the constants,) both terms are 'length^3', giving volume.	E1	
	3	

2. *November 2008 Paper 1 Higher*

1 3 1 2	B2 2	For all 4 correct. B1 for any 3 correct OR B1 for all 4 dimensions implied by the indices in, for example, km, m^3, cm, cm^2.

3. *June 2005 Paper 1 Higher*

length none of these volume area	B2 2	For all 4 correct. B1 for any 3 correct.

4. *June 1998 Linear Intermediate*

(a) Explanation that the expression (on the right) is for volume OR is three-dimensional	E1	
(b) $L = 5W + 4D + 4H.$	B1	
(Disregarding the constants,) both terms are 'length^3', giving volume.	E1	
	3	

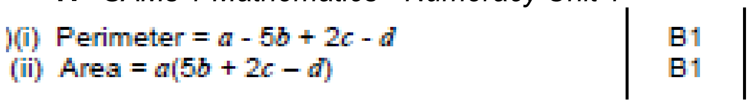
5. *June 2008 Paper 1 Higher*

volume length none of these area	B2 2	For all 4 correct. B1 for any 3 correct.

6. *June 2007 Paper 1 Higher*

1 2 1 3	B2 2	For all 4 correct. B1 for any 3 correct OR B1 for all 4 dimensions implied by the indices in, for example, km, m^2, cm, cm^3.

7. *SAMs 1 Mathematics - Numeracy Unit 1*



)(i) Perimeter = $a - 5b + 2c - d$	B1	
(ii) Area = $a(5b + 2c – d)$	B1	

3.5 POPULATION DENSITY

Specification statement (Intermediate and Higher tiers, Mathematics - Numeracy and Mathematics)

Using compound measures including speed, density and population density.
Using compound measures such as m/s, km/h, mph, mpg, kg/m^3, g/cm^3, population per km^2.

Notes

Population density is a measurement of population per unit.
The term 'population' could extend to beyond 'number of people' e.g.

- number of houses per square mile
- number of bacteria per cubic mm,
- number of snails per square metre.

Example

The table below shows the land area (square kilometre) and population of all 22 Welsh local authorities in 2013.
https://statswales.wales.gov.uk/Catalogue/Population-and-Migration/Population/Density/PopulationDensity-by-LocalAuthority-Year

Mid-year 2013	Land area (km^2)	Population
Wales	20735.5	3 082 412
Blaenau Gwent	108.7	69 789
Bridgend	250.7	140 480
Caerphilly	277.4	179 247
Cardiff	140.4	351 710
Carmarthenshire	2370.3	184 681
Ceredigion	1785.6	75 964
Conwy	1125.8	115 835
Denbighshire	836.7	94 510
Flintshire	437.5	153 240
Gwynedd	2534.9	121 911
Isle of Anglesey	711.3	70 091
Merthyr Tydfil	111.4	59 021
Monmouthshire	849.1	92 100
Neath Port Talbot	441.3	139 898
Newport	190.5	146 558
Pembrokeshire	1618.7	123 261
Powys	5180.7	132 705
Rhondda Cynon Taf	424.2	236 114
Swansea	379.7	240 332
Torfaen	125.7	91 407
Vale of Glamorgan	330.9	127 159
Wrexham	503.8	136 399

Possible questions:

- Which authorities are the most crowded?
- Will the authority with the largest population be the most populous?
- What is the best way of comparing each authority?
- How would you calculate how many people, on average, live in each 1km^2?

Example of an examination question on population density

SAMs 1 Mathematics – Numeracy Unit 2 Higher

The following table gives areas and populations of 6 countries.

Country	**Area (km^2)**	**Population in 2014**
Wales	20 761	3 006 000
Singapore	716	5 399 200
Bermuda	53	64 237
India	3 287 240	1 244 392 079
Belgium	30 528	11 194 824
Tonga	720	104 270

(a) How many times as dense is the country with the greatest population density as the country with the least population density?
You must show all your working. [4]

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(b) Which two countries have the same population densities to the nearest whole number of people per km^2? [1]

Circle your answer.

India and Belgium	Wales and Tonga	Singapore and Tonga	Wales and Belgium	Bermuda and Tonga

(c) If the information in the table had all been given correct to 2 significant figures would this make a difference to your answer in part *(a)*? [2]

Circle either TRUE or FALSE for each of the following statements.

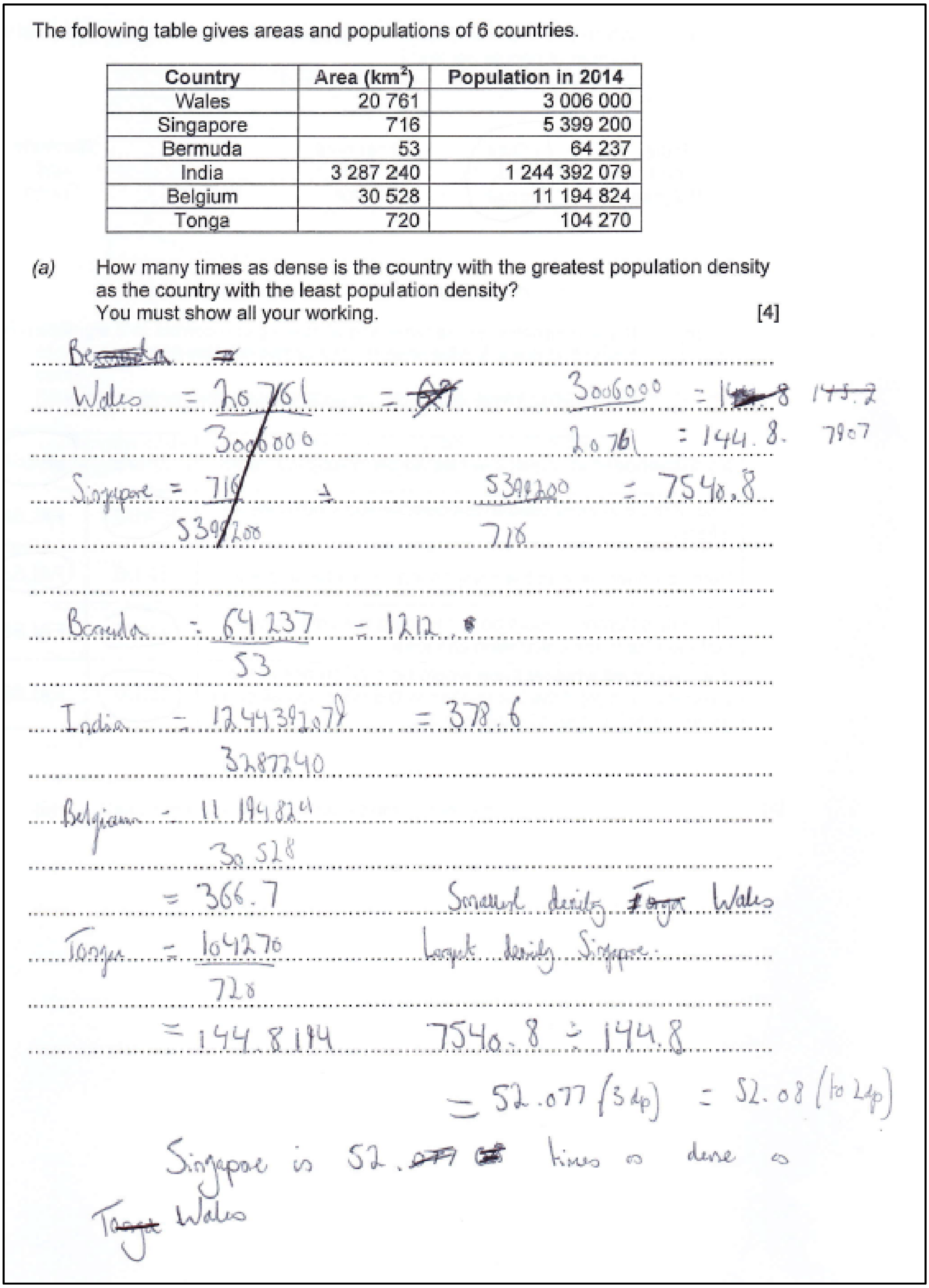
No difference at all, the answer would be exactly the same.	TRUE	FALSE
One of the countries used in the comparison would be different.	TRUE	FALSE
Both countries used in the comparison would be different.	TRUE	FALSE
The only difference would be in rounding the final answer, nothing else in the calculation changes.	TRUE	FALSE
You cannot tell whether there would be a difference in the answer in part *(a)* if the information in the table had all been given correct to 2 significant figures.	TRUE	FALSE

Mark scheme

<table>
<tr><td>(a) Correct or reasonable estimates for the population densities, identifying Singapore as greatest and Wales as the least.</td><td>B2</td><td>Singapore and Wales may not be identified explicitly but implied in later working.
B1 at least 3 reasonable estimates for the population densities
<table><tr><td>Country</td><td>Population density</td></tr><tr><td>Wales</td><td>144.790713…</td></tr><tr><td>Singapore</td><td>7540.78..</td></tr><tr><td>Bermuda</td><td>1212.018…</td></tr><tr><td>India</td><td>378.55..</td></tr><tr><td>Belgium</td><td>366.706…</td></tr><tr><td>Tonga</td><td>144.819..</td></tr></table></td></tr>
<tr><td>7540.78 ÷ 144.790713…</td><td>M1</td><td></td></tr>
<tr><td>52(.0805…. times)</td><td>A1</td><td></td></tr>
<tr><td>(b) Wales and Tonga</td><td>B1</td><td></td></tr>
<tr><td>(c) False
True
False
False
False</td><td>B2</td><td>B1 for 4 correct</td></tr>
<tr><td></td><td>7</td><td></td></tr>
</table>

Candidate responses (part *(a)* only)

Candidate A



The following table gives areas and populations of 6 countries.

Country	Area (km^2)	Population in 2014
Wales	20 761	3 006 000
Singapore	716	5 399 200
Bermuda	53	64 237
India	3 287 240	1 244 392 079
Belgium	30 528	11 194 824
Tonga	720	104 270

(a) How many times as dense is the country with the greatest population density as the country with the least population density?
You must show all your working. **[4]**

~~Bermuda~~ ~~=~~

Wales ~~$= \frac{20761}{3006000} =$~~ $\frac{3006000}{20761} = 144.8$ ~~144.8~~ ~~145.2~~ 7907

Singapore ~~$= \frac{716}{5399200}$~~ $\frac{5399200}{716} = 7540.8$

Bermuda $= \frac{64237}{53} = 1212$

India $= \frac{1244392078}{3287240} = 378.6$

Belgium $= \frac{11194824}{30528}$

$= 366.7$

Smallest density ~~Tonga~~ Wales

Largest density Singapore.

Tonga $= \frac{104270}{720}$

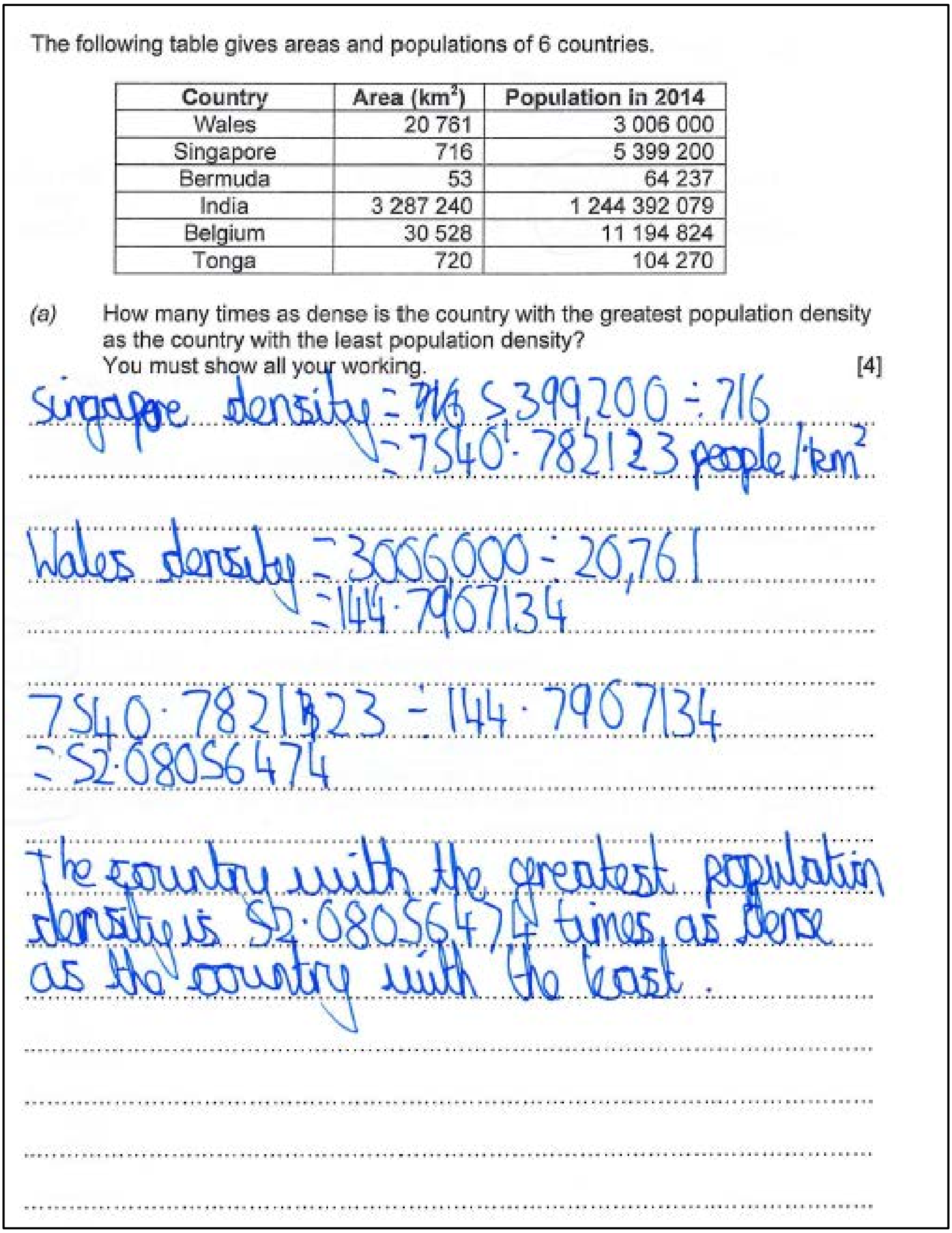
$= 144.8194$

$7540.8 \div 144.8$

$= 52.077$ (3dp) $= 52.08$ (to 2dp)

Singapore is 52.~~077~~ ~~08~~ times as dense as ~~Tonga~~ Wales

Candidate B



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Bermuda	53	64 237
India	3 287 240	1 244 392 079
Belgium	30 528	11 194 824
Tonga	720	104 270

(a) How many times as dense is the country with the greatest population density as the country with the least population density?
You must show all your working. [4]

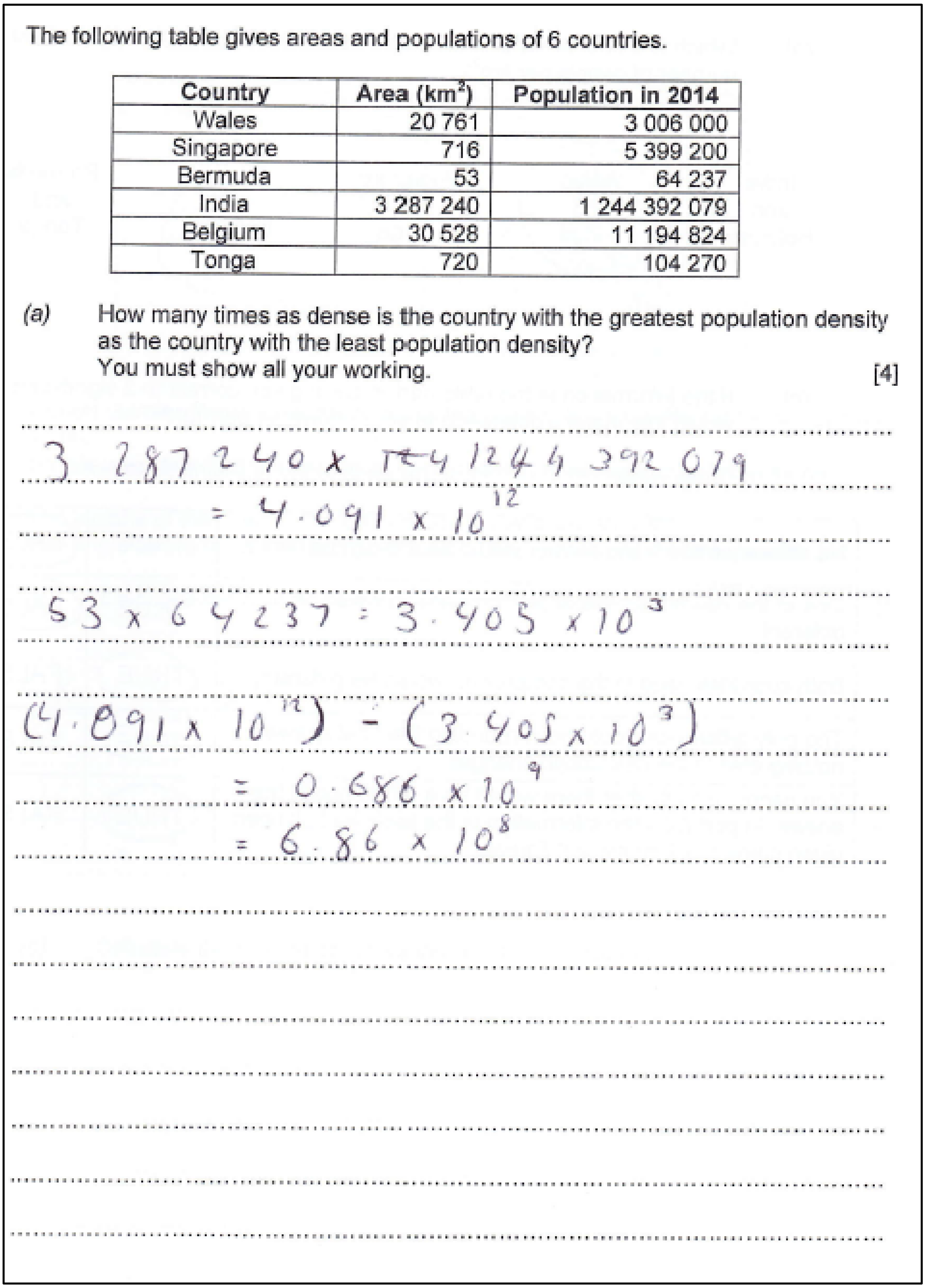
Singapore density = ~~716~~ 5,399,200 ÷ 716
= 7540·782123 people /km^2

Wales density = 3006000 ÷ 20,761
= 144·7967134

7540·782123 ÷ 144·7967134
= 52·08056474

The country with the greatest population density is 52·08056474 times as dense as the country with the least.

Candidate C



The following table gives areas and populations of 6 countries.

Country	Area (km²)	Population in 2014
Wales	20 761	3 006 000
Singapore	716	5 399 200
Bermuda	53	64 237
India	3 287 240	1 244 392 079
Belgium	30 528	11 194 824
Tonga	720	104 270

(a) How many times as dense is the country with the greatest population density as the country with the least population density?
You must show all your working. [4]

3 287 240 x ~~124~~ 1244 392 079

$= 4.091 \times 10^{12}$

$53 \times 64237 = 3.405 \times 10^{3}$

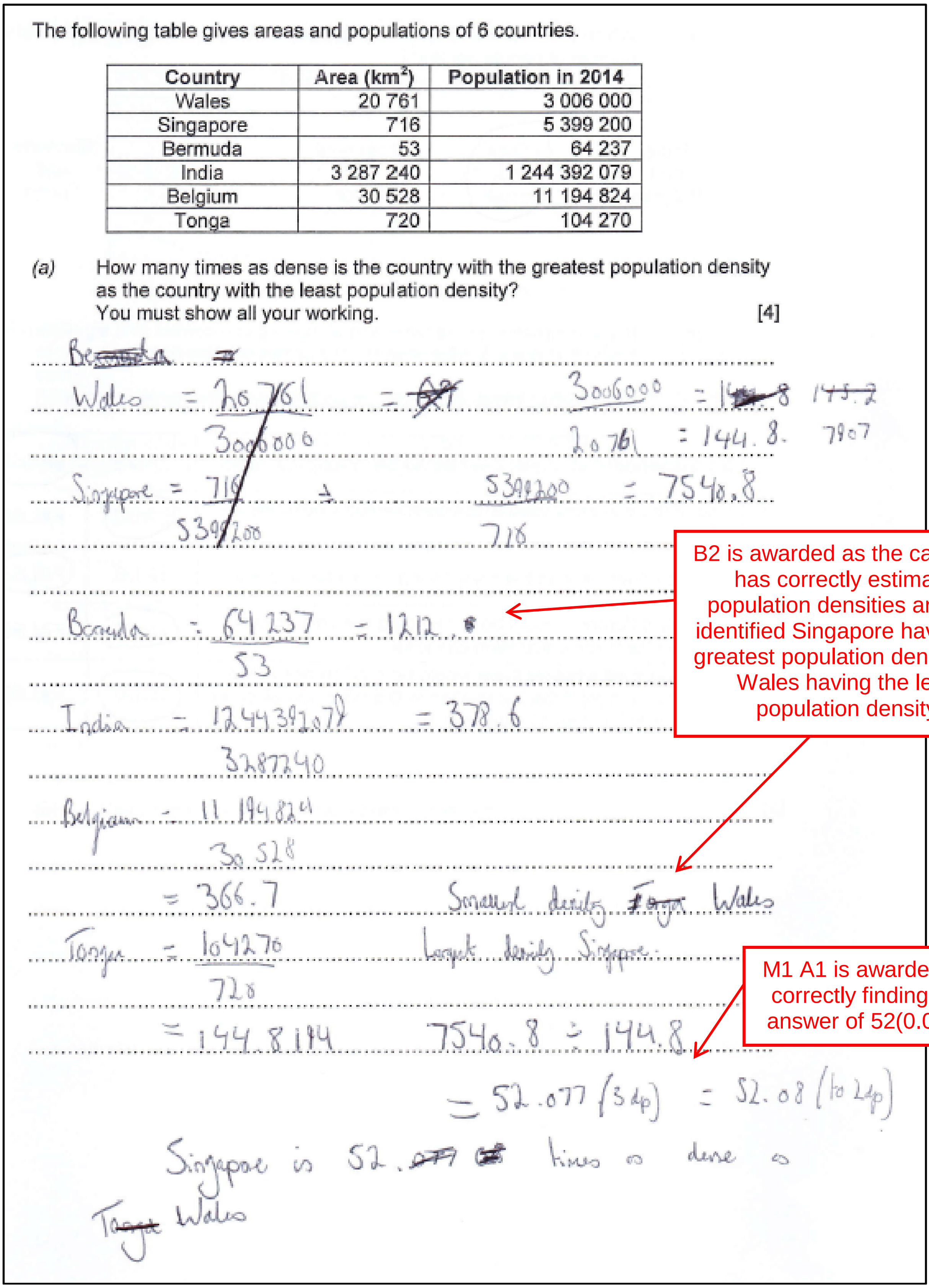
$(4.091 \times 10^{12}) \div (3.405 \times 10^{3})$

$= 0.686 \times 10^{9}$

$= 6.86 \times 10^{8}$

Annotated candidate responses

Candidate A



The following table gives areas and populations of 6 countries.

Country	Area (km^2)	Population in 2014
Wales	20 761	3 006 000
Singapore	716	5 399 200
Bermuda	53	64 237
India	3 287 240	1 244 392 079
Belgium	30 528	11 194 824
Tonga	720	104 270

(a) How many times as dense is the country with the greatest population density as the country with the least population density?
You must show all your working. [4]

~~Bermuda~~ ~~=~~

Wales $= \cancel{\frac{20761}{3006000}} = \cancel{69}$ $\frac{3006000}{20761} = 1\cancel{44}.8$ ~~145.2~~ $= 144.8.$ 7907

Singapore $= \cancel{\frac{716}{5399200}}$ ÷ $\frac{5399200}{716} = 7540.8$

Bermuda $= \frac{64237}{53} = 1212.$

India $= \frac{1244392079}{3287240} = 378.6$

Belgium $= \frac{11194824}{30528}$

$= 366.7$ Smallest density ~~Tonga~~ Wales

Tonga $= \frac{104270}{720}$ Largest density Singapore.

$= 144.8194$ $7540.8 \div 144.8$

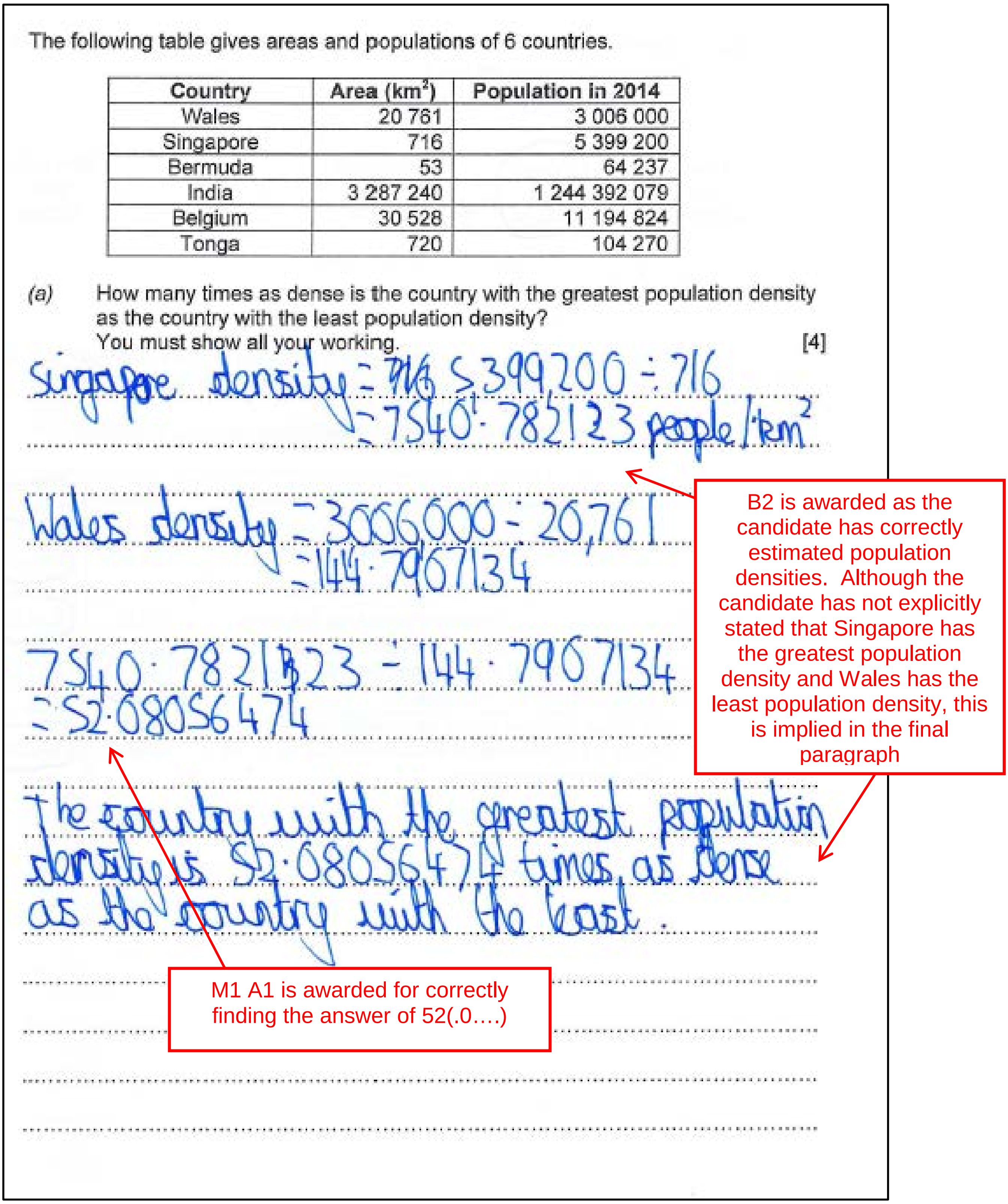
$= 52.077$ (3dp) $= 52.08$ (to 2dp)

Singapore is 52.~~077~~ ~~08~~ times as dense as ~~Tonga~~ Wales

B2 is awarded as the candidate has correctly estimated population densities and has identified Singapore having the greatest population density and Wales having the least population density.

M1 A1 is awarded for correctly finding the answer of 52(0.0….)

Candidate B



The following table gives areas and populations of 6 countries.

Country	Area (km^2)	Population in 2014
Wales	20 761	3 006 000
Singapore	716	5 399 200
Bermuda	53	64 237
India	3 287 240	1 244 392 079
Belgium	30 528	11 194 824
Tonga	720	104 270

(a) How many times as dense is the country with the greatest population density as the country with the least population density?
You must show all your working. [4]

Singapore density = ~~716~~ 5,399,200 ÷ 716
= 7540.782123 people/km^2

Wales density = 3006000 ÷ 20,761
= 144.7967134

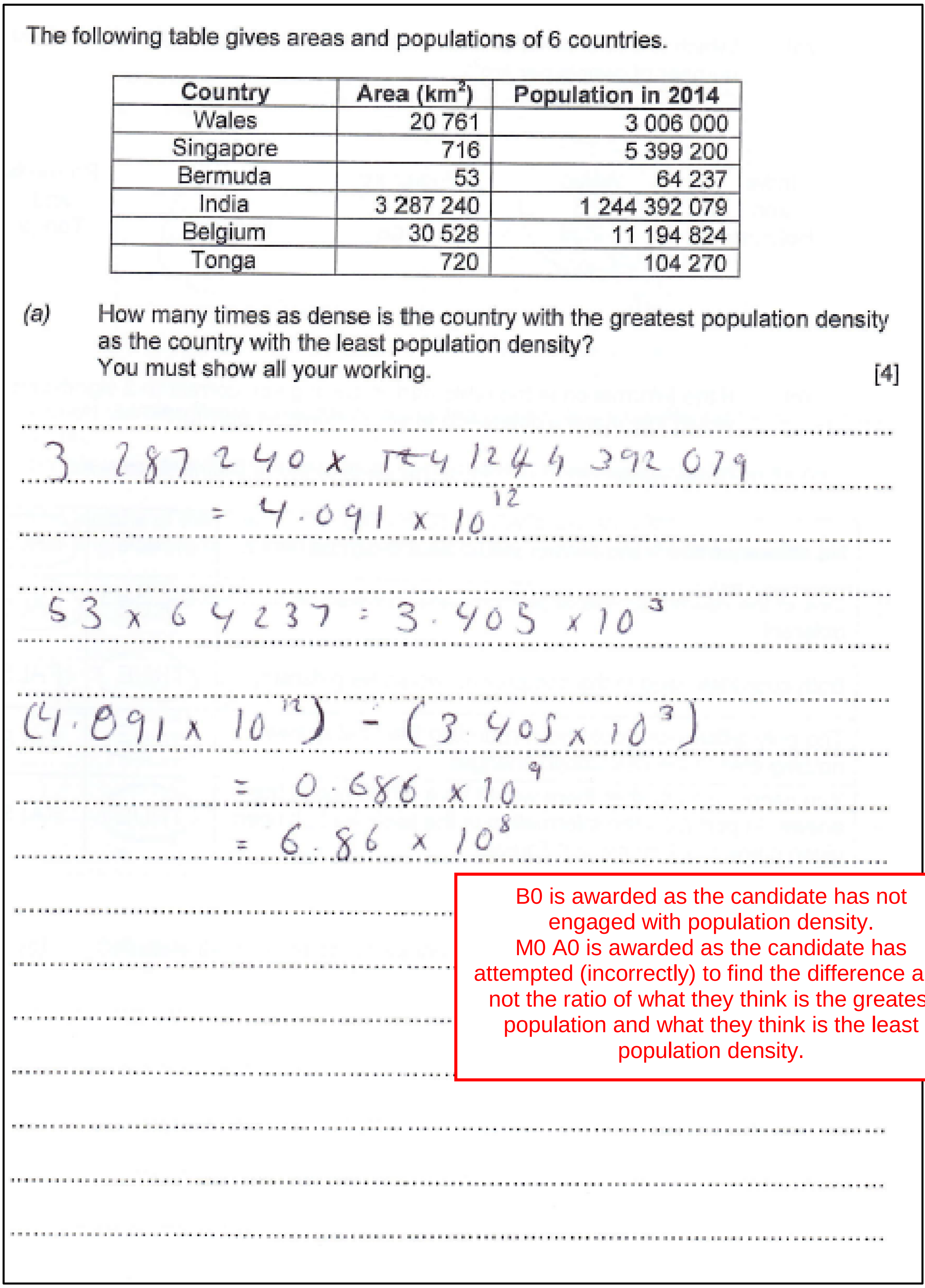
7540.782123 ÷ 144.7967134
= 52.08056474

The country with the greatest population density is 52.08056474 times as dense as the country with the least.

B2 is awarded as the candidate has correctly estimated population densities. Although the candidate has not explicitly stated that Singapore has the greatest population density and Wales has the least population density, this is implied in the final paragraph

M1 A1 is awarded for correctly finding the answer of 52(.0….)

Candidate C



The following table gives areas and populations of 6 countries.

Country	Area (km^2)	Population in 2014
Wales	20 761	3 006 000
Singapore	716	5 399 200
Bermuda	53	64 237
India	3 287 240	1 244 392 079
Belgium	30 528	11 194 824
Tonga	720	104 270

(a) How many times as dense is the country with the greatest population density as the country with the least population density?
You must show all your working. [4]

$3\ 287\ 240 \times$ ~~124~~ $1244\ 392\ 079$

$= 4.091 \times 10^{12}$

$53 \times 64\ 237 = 3.405 \times 10^3$

$(4.091 \times 10^{12}) - (3.405 \times 10^3)$

$= 0.686 \times 10^9$

$= 6.86 \times 10^8$

B0 is awarded as the candidate has not engaged with population density.
M0 A0 is awarded as the candidate has attempted (incorrectly) to find the difference and not the ratio of what they think is the greatest population and what they think is the least population density.

3.6 TRANSLATION (expressed as a vector)

Specification statement (Intermediate and Higher tiers, Mathematics only)

Description of translations using column vectors.

Notes

A **column vector** is an efficient way to describe a translation.

It is written as $\begin{pmatrix} a \\ b \end{pmatrix}$,

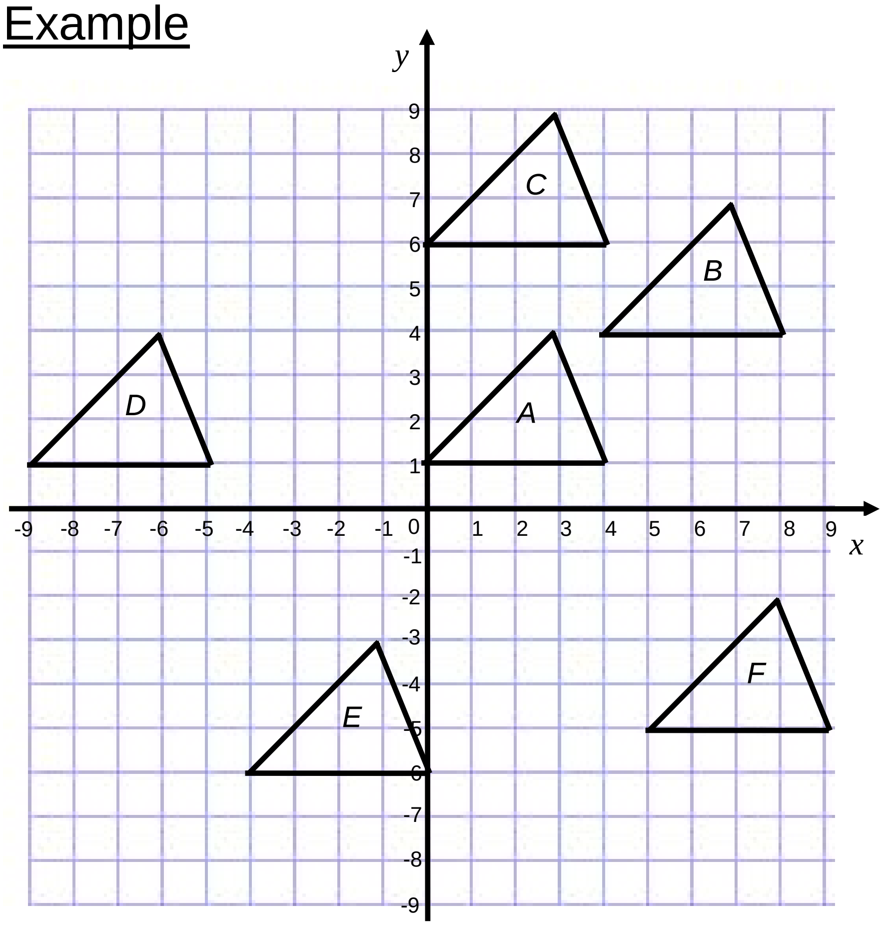
where *a* denotes the horizontal distance travelled and *b* denotes the vertical distance travelled.

Notice that there is no horizontal line between *a* and *b*. It should not look like a fraction.

The brackets are curved, not straight or square.

If a shape moves to the right, *a* is positive. If it moves to the left, *a* is negative.
If a shape moves up, *b* is positive. If it moves down, *b* is negative.

Example

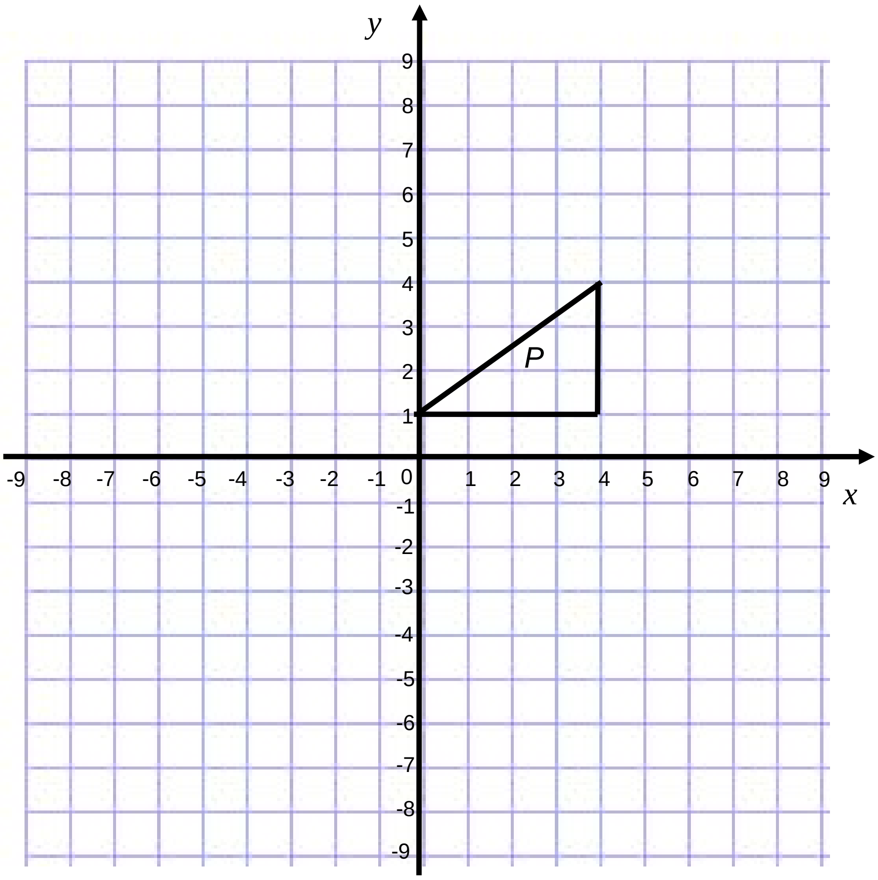


Triangle *A* is translated using 5 different vectors, as given in the table below.

Translation	Vector
A to *B*	$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$
A to *C*	$\begin{pmatrix} 0 \\ 5 \end{pmatrix}$
A to *D*	$\begin{pmatrix} -9 \\ 0 \end{pmatrix}$
A to *E*	$\begin{pmatrix} -4 \\ -7 \end{pmatrix}$
A to *F*	$\begin{pmatrix} 5 \\ -6 \end{pmatrix}$

Examples of questions on translations

1.



(a) Translate triangle *P* using the vector $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$. Label the image *Q*.

[1]

(b) Translate triangle *P* using the vector $\begin{pmatrix} -8 \\ 0 \end{pmatrix}$. Label the image *R*.

[1]

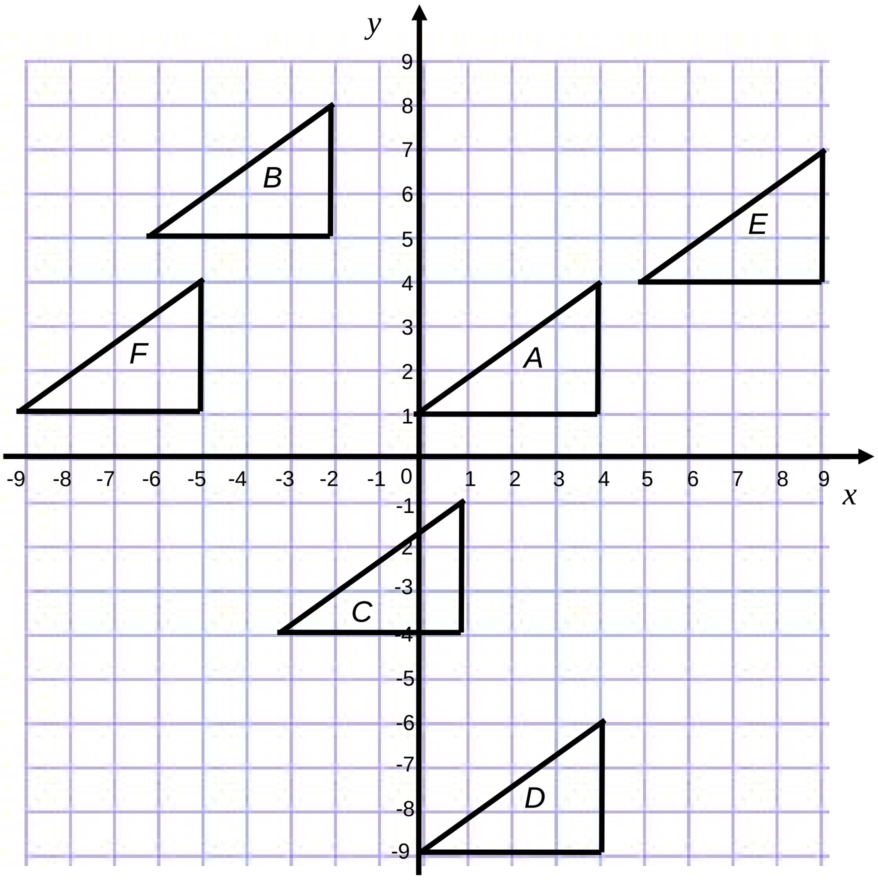
(c) Translate triangle *P* using the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Label the image *S*.

[1]

(d) Translate triangle *P* using the vector $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$. Label the image *T*.

[1]

2.



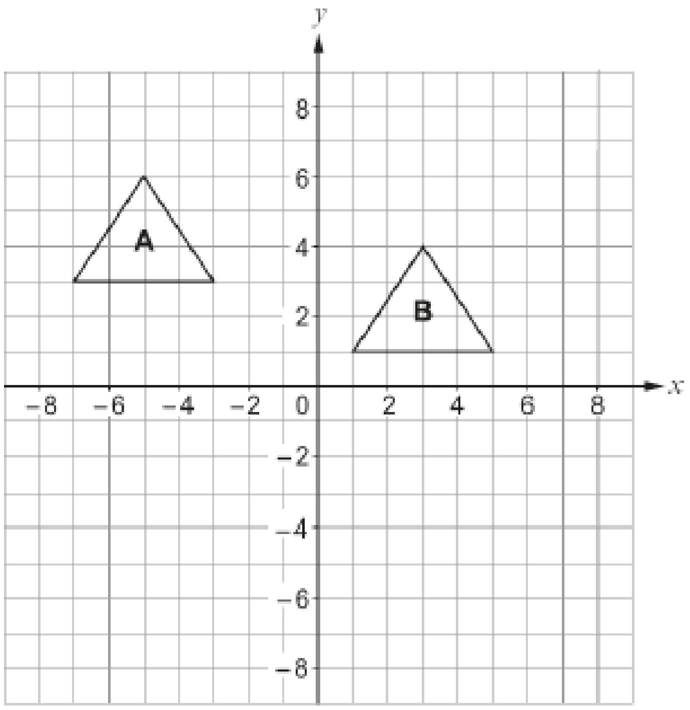
Triangle *A* is translated using 5 different vectors. Complete the table.

[5]

Translation	Vector
A to *B*	
A to *C*	
A to *D*	
A to *E*	
A to *F*	

3. *SAMs 1 Mathematics Unit 1 Intermediate and Higher*

Shape A is translated onto Shape B.



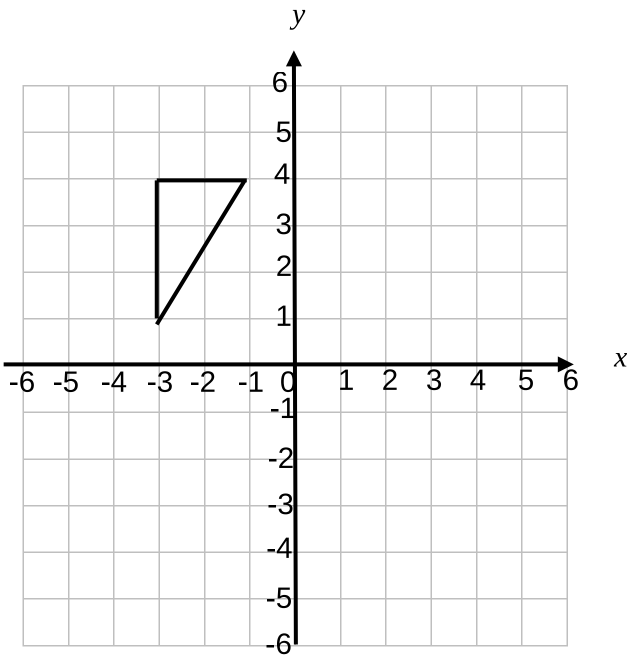
Which one of the following vectors describes the translation?
Circle your answer. [1]

$\begin{pmatrix} 8 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 2 \\ -8 \end{pmatrix}$ $\begin{pmatrix} -8 \\ -2 \end{pmatrix}$ $\begin{pmatrix} -2 \\ 8 \end{pmatrix}$ $\begin{pmatrix} -8 \\ 2 \end{pmatrix}$

4. *SAMs 2 Mathematics Unit 2 Intermediate*

Translate the triangle using the column vector $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$.

[1]



(ii) Write down the column vector that will **reverse** the translation in part (i).

[1]

………………………………………………………………………………………………………

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………………………………………………………………………………………………………

Mark schemes for examination questions on translations

1.

(a) Correct translation *(b)* Correct translation *(c)* Correct translation *(d)* Correct translation *(e)* Correct translation	B1 B1 B1 B1 B1 5	

2.

A to *B* $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$	B1	
A to *C* $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$	B1	
A to *D* $\begin{pmatrix} 0 \\ -10 \end{pmatrix}$	B1	
A to *E* $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$	B1	
A to *F* $\begin{pmatrix} -9 \\ 0 \end{pmatrix}$	B1	
	5	

3. *SAMs 1 Mathematics Unit 1 Intermediate and Higher*

$\begin{pmatrix} 8 \\ -2 \end{pmatrix}$	B1	

4. *SAMs 2 Intermediate Unit 2 Mathematics*

(c) (i) Correct translation (ii) $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$	B1 B1	B1 for correctly sized rectangle in incorrect position OR consistent use of wrong scale factor OR 2 correct vertices

Further examples of questions can be found on the WJEC website in Unit 2 Methods in Mathematics papers (4364/01 and 4364/02) from January 2011 onwards (January and June series).

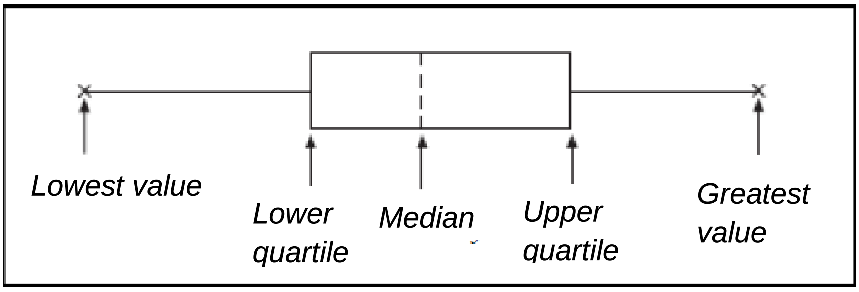
3.7 BOX-AND-WHISKER PLOTS

Specification statement (Intermediate and Higher tiers, Mathematics - Numeracy and Mathematics)

Producing and using box-and-whisker plots to compare distributions.

Notes

A box-and-whisker plot is a graphical display which shows certain summary statistics.
The left and right edges of the rectangle indicate the lower and upper quartiles.
The median is marked across the body of the box.
Whiskers extend from the ends of the box to show the lowest and greatest values of the distribution.



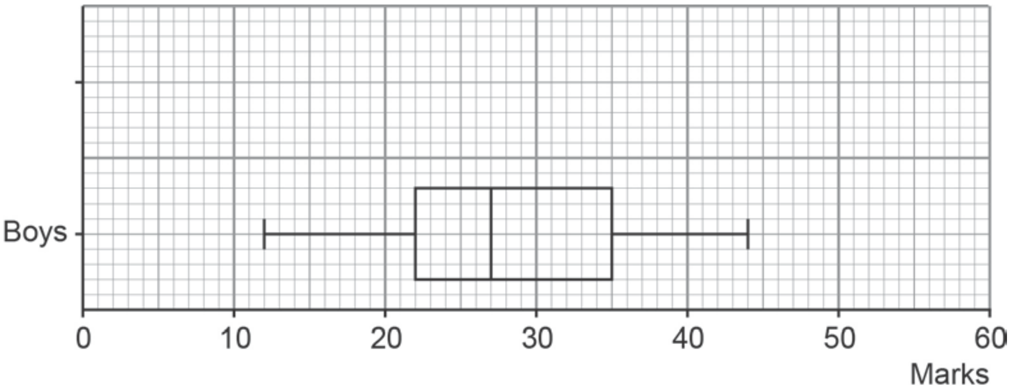
Box-and-whisker plots are especially useful when you want to compare two distributions.
Box-and-whisker plots can be drawn either horizontally or vertically.

Possible extension ideas

- Skewness of distributions.
- Potential outliers.

Example

Some boys and girls sit a maths test.
The box plot shows information about the boys' results.



The table shows information about the **girls'** results.

Girls' results

Minimum	Lower quartile	Median	Upper quartile	Maximum
15	25	38	42	50

(a) On the graph paper above, use this data to draw a box-and-whisker plot to show the distribution of the girls' results.

[3]

(b) Compare the distributions of the results of the boys' and girls'.

[2]

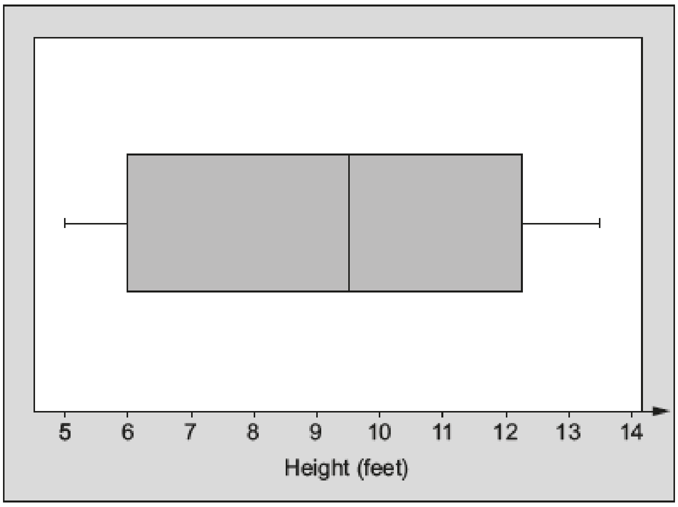
(a) (b) Comparison referring to central tendency or comparative size Comparison of spread	B3 E1 E1 5	B1 for range ends 15 and 50 correctly indicated with 'whiskers' B1 for median line correctly indicated B1 for LQ and UQ correctly indicated Accept reference to skewness

Notes:
In this example the height of the Girls' box-and-whisker plot should be the same height as that of the Boys. However, pupils will not be penalised if not.
When asked to compare distributions using box-and whisker plots, marks will be awarded for comparing central tendency (e.g. medians) or comparative size and the spread (e.g. interquartile range or the range).

Examples of examination questions on box-and-whisker plots

1. *SAMs 1 Mathematics – Numeracy Unit 1 Intermediate and Higher*

The box-and-whisker plot shows information about the height, in feet, of waves measured at a beach on a particular day.



(a) About what fraction of the waves measured were less than 6 feet? [1]

……………………………………………………………………………………………………

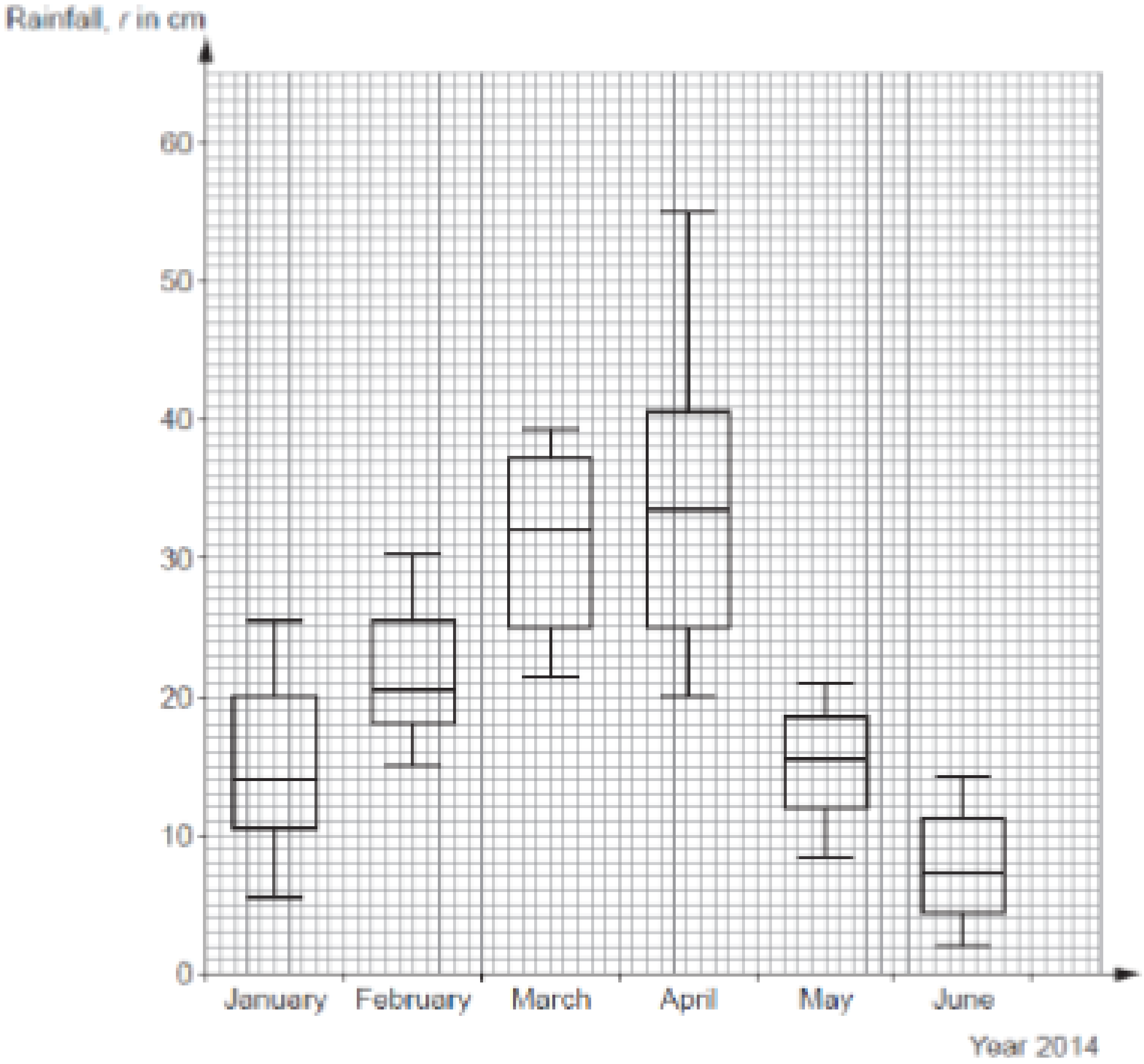
……………………………………………………………………………………………………

(b) Circle either TRUE or FALSE for each of the following statements. [2]

The smallest wave measured was 5 feet.	TRUE	FALSE
The range of the heights of the waves measured was 6·5 feet.	TRUE	FALSE
Approximately a half of the waves measured were more than 9·5 feet.	TRUE	FALSE
Approximately a quarter of the waves measured were between 6 feet and 9·5 feet.	TRUE	FALSE
The biggest wave measured was 12·25 feet.	TRUE	FALSE

2. *SAMs 2 Mathematics – Numeracy Unit 1 Intermediate and Higher*

The information shown below was found in a holiday brochure for a small island.



The information shows monthly data about the rainfall in centimetres.

(a) Looking at the rainfall, which month had the most changeable weather?
You must give a reason for your answer.

[1]

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……………………………………………………………………………………………

(b) Circle either TRUE or FALSE for each of the following statements.

[2]

If you don't want much rain, the time to visit the island is in June.	TRUE	FALSE
The greatest difference in rainfall is between the months of February and March	TRUE	FALSE
The interquartile range for May is approximately equal to the interquartile range for June.	TRUE	FALSE
The range of rainfall in February was approximately 15 cm.	TRUE	FALSE
During June, there were more days with greater than 7·5 cm of rainfall than there were days with less than 7·5 cm of rainfall.	TRUE	FALSE

(c) In July 2014, the interquartile range for the rainfall was 10 cm and the range was 40 cm.
Is it possible to say whether July has more or less rainfall than June?
You must give a reason for your answer.

[1]

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……………………………………………………………………………………………

……………………………………………………………………………………………

Mark schemes for examination questions on box-and-whisker plots

1. *SAMs 1 Mathematics – Numeracy Unit 1 Intermediate and Higher*

(a) ¼ or equivalent	B1	
(b) TRUE FALSE TRUE TRUE FALSE	B2	B1 for any 4 correct
	3	

2. *SAMs 2 Mathematics – Numeracy Unit 1 Intermediate and Higher*

(a) April Reason, e.g. greatest range, or greatest interquartile range	E1	
(b) TRUE FALSE TRUE TRUE FALSE	B2	B1 for any 4 correct.
(c) State or implies 'not possible to tell' with a reason, e.g. 'can't tell as it doesn't give any information about how much rain fell', or 'just the difference between maximum and minimum not how much rain fell', or 'don't know as the difference between UQ & LQ doesn't give the actual amount of rain, just a range for the middle 50%'.	B1	
	4	

Further examples of questions can be found on the WJEC website in Unit 1 Higher Applications of Mathematics papers (4361/02) from January 2011 onwards (January and June series).

3.8 SAMPLING

Specification statement (Intermediate and Higher tiers, Mathematics - Numeracy and Mathematics)

Specifying the data needed and considering potential sampling methods.
Sampling systematically
Working with stratified sampling techniques and defining a random sample.

A. Stratified sampling (*stratum* = layer)

Notes

For stratified sampling, the population is divided into groups which have something in common e.g. school year groups. The number selected from each of these groups will be proportional to the size of the group.

Example

Bethan needs to survey 50 pupils from her school in order to gather opinions on school uniform. The numbers in each year group are given in the table.

Year group	7	8	9	10	11
Number of pupils	242	209	203	178	160

Calculate the number of pupils she should select from each year group.

Solution

Total number of pupils = 242 + 209 + 203 + 178 + 160 = 992

Number from Year 7 $= \frac{242}{992} \times 50 = 12{\cdot}20$ Answer = 12 Year 7 pupils

Number from Year 8 $= \frac{209}{992} \times 50 = 10{\cdot}53$ Answer = 11 Year 8 pupils

Number from Year 9 $= \frac{203}{992} \times 50 = 10{\cdot}23$ Answer = 10 Year 9 pupils

Number from Year 10 $= \frac{178}{992} \times 50 = 8{\cdot}97$ Answer = 9 Year 10 pupils

Number from Year 11 $= \frac{160}{992} \times 50 = 8{\cdot}06$ Answer = 8 Year 11 pupils

It is important to check the total 12 + 11 + 10 + 9 + 8 = 50 as rounding the individual answers can sometimes lead to a different total (in which case 1 more or less need to be taken from a specific group).

B. Random sampling

Notes

For a random sample,
every member of the population has an equal chance of being selected.

Possible methods include picking names out of a hat or using random numbers (from a published table or from a calculator).

Example

The following list of random numbers was produced by using the random number button (RND) on a calculator. (All the digits were equally likely to be selected and were independent of each other.)

139 508 680 812 562 240 442 389 210 964 670 373 797 488 055

We can use these numbers to randomly select a sample of 5 people out of 80.

Firstly, number all the people from 1 to 80.
Read the random digits in pairs to produce 2-digit numbers (13, 95, 08, 68, ...).
Write down the first 5 of these that are 80 or less (ignore 00 or numbers greater than 80 or repeats).

The 5 selected people are those numbered 13, (0)8, 68, 12, 56.

C. Systematic sampling

Updated guidance on Systematic Sampling

Systematic sampling is a sampling method in which sample members from a population are selected using a random starting point and a fixed interval.

Systematic sampling involves taking one item from a list at regular fixed intervals e.g. every 5th, every 20th, etc.

It is useful in certain situations e.g. in regularly testing the quality of items manufactured in a factory.

The sampling is started by first selecting an item from the list at random, and then every k^{th} item is selected, where k, the sampling interval, is calculated as

$$k = \frac{N}{n},$$

where n is the sample size, and N is the population size.

For example, to select a systematic sample of 10 items from 120, you work out the sampling interval as $k = 120 \div 10$
$k = 12$.
You then number the items from 1 to 120.
You then pick one item at random, e.g. the 5th. Note: It is easier to pick one from the first 12.
You then pick every 12th from there on.
Therefore the sample will be the following items: 5, 17, 29, 41, 53, 65, 77, 89, 101, 113.

If you were to start on 17, instead of 5, you would get the same sample, just in a different order (17, 29, 41, 53, 65, 77, 89, 101, 113, 5).
Note: the '5' is obtained by counting 7 from 113 to 120 and then 5 from 1 to 5.

The sample you get is considered to be a random sample, since every item has an equal probability of being chosen.
However, the difference between systematic sampling and simple random sampling is that in systematic sampling not every possible sample of a certain size has an equal chance of being chosen.

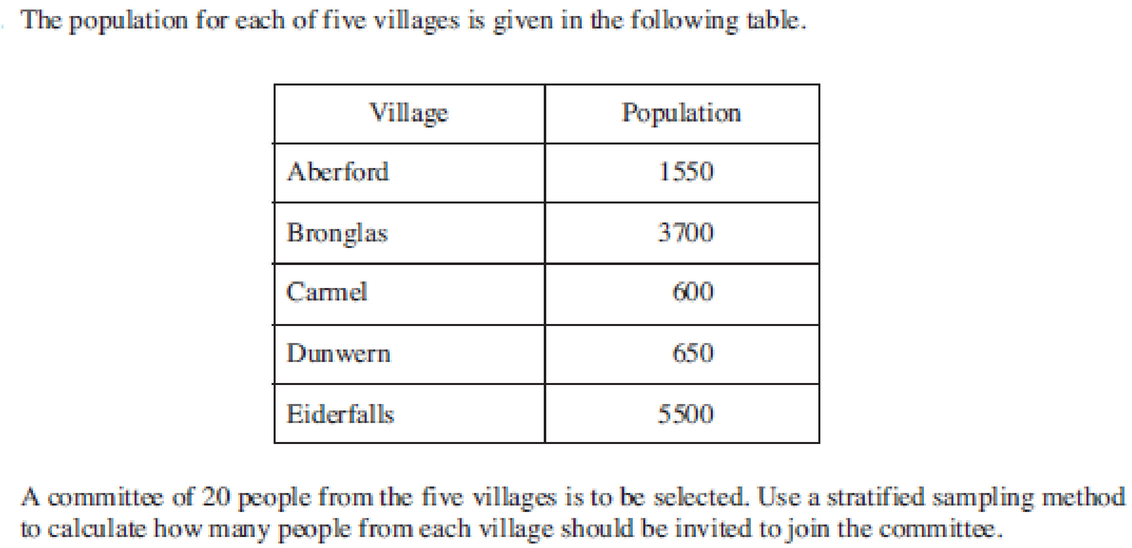
For example, in the above case of a systematic sample of 10 items from 120, there is an equal

chance of	6, 18, 30, 42, 54, 66, 78, 90 102, 114	being chosen
as there is a chance of	5, 17, 29, 41, 53, 65, 77, 89, 101, 113	being chosen.
However, there is no chance of	5, 8, 23, 28, 56, 79, 101, 102, 113, 118	being chosen.

Note that if the population is not a multiple of the sample size required (which is usually the case) then, in order to ensure that every item has the same probability of being selected, the first item should be selected at random from the whole population, and the sample should be generated by cycling back to the start of the population when the end is reached.

Examples of examination questions on stratified sampling

1. *June 2006 Paper 2 Higher*



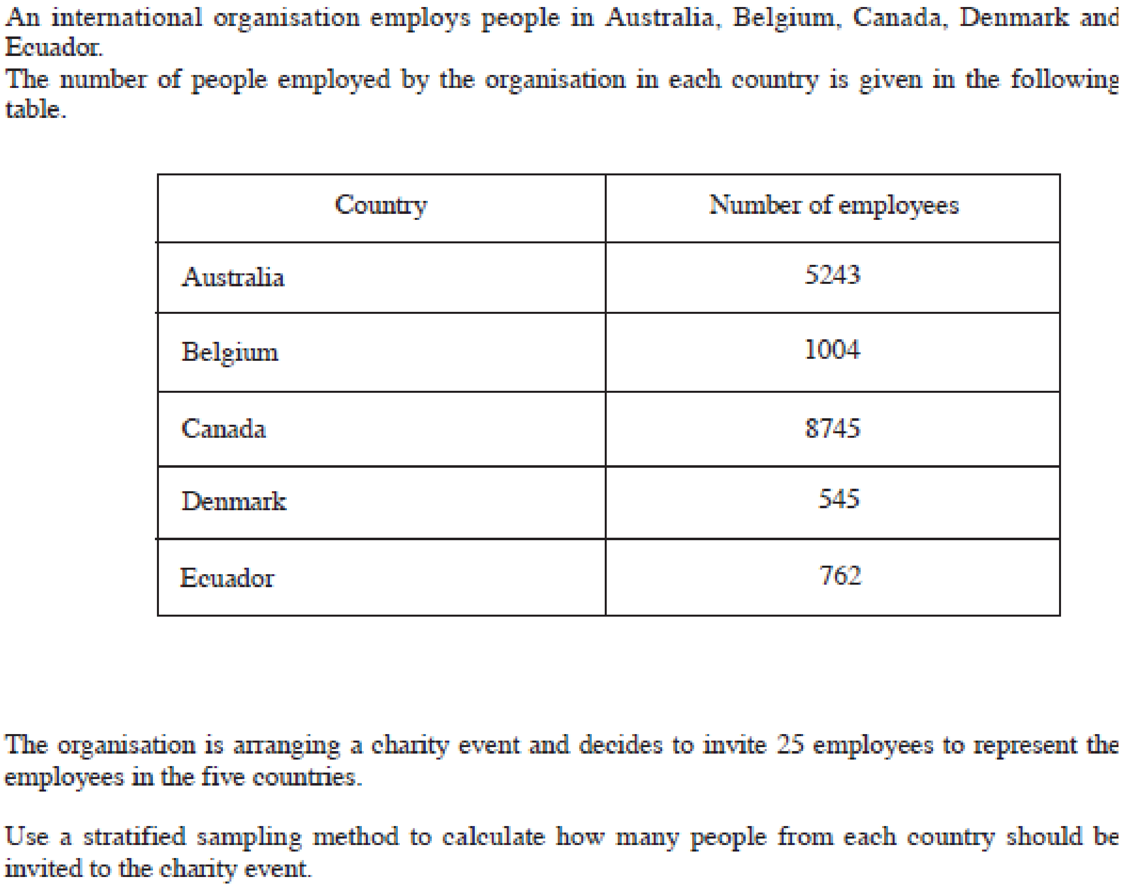
The population for each of five villages is given in the following table.

Village	Population
Aberford	1550
Bronglas	3700
Carmel	600
Dunwern	650
Eiderfalls	5500

A committee of 20 people from the five villages is to be selected. Use a stratified sampling method to calculate how many people from each village should be invited to join the committee.

[4]

2. *June 2007 Paper 2 Higher*



An international organisation employs people in Australia, Belgium, Canada, Denmark and Ecuador.
The number of people employed by the organisation in each country is given in the following table.

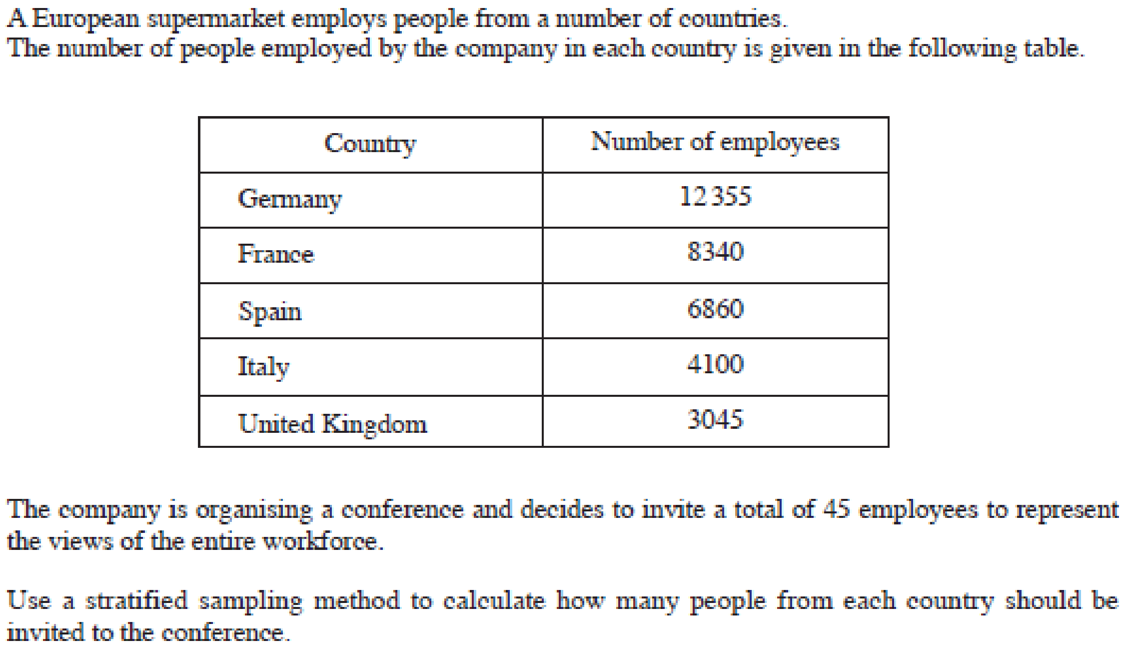
Country	Number of employees
Australia	5243
Belgium	1004
Canada	8745
Denmark	545
Ecuador	762

The organisation is arranging a charity event and decides to invite 25 employees to represent the employees in the five countries.

Use a stratified sampling method to calculate how many people from each country should be invited to the charity event.

[4]

3. *June 2008 Paper 2 Higher*



A European supermarket employs people from a number of countries.
The number of people employed by the company in each country is given in the following table.

Country	Number of employees
Germany	12 355
France	8340
Spain	6860
Italy	4100
United Kingdom	3045

The company is organising a conference and decides to invite a total of 45 employees to represent the views of the entire workforce.

Use a stratified sampling method to calculate how many people from each country should be invited to the conference.

[4]

4. *June 1996 Higher*

The governors of a school are planning to open the school swimming pool for public use at certain times when the school is closed. They want to know how many pupils and their families are prepared to pay to use the pool if it is open at weekends and during the school holidays. They have prepared a questionnaire on the subject for pupils to take home and complete with their families and want to select a sample of pupils for this purpose.

Write down a factor that you think they should take into account when constructing a stratified sample for this survey, explaining why you have chosen that factor.

[2]

5. *June 2000 Higher*

A survey of cars was carried out. It was noted whether the cars were up to 3 years old inclusive or over 3 years old. It was also noted whether the cars had a diesel engine or a petrol engine. The results of the survey were as follows.

	Diesel engine	Petrol engine
Up to 3 years old (inclusive)	190	650
Over 3 years old	260	900

Use this information to estimate how many cars with diesel engines you would expect to find in a county known to have 40 000 cars.

[3]

6. *SAMs 2 Mathematics - Numeracy Unit 1 Higher*

(a) At the National Eisteddfod in August each year, a concert is performed on the opening night.

Of those performing this year:

- 39 are primary school children,
- 73 are secondary school children,
- 128 are adults.

In order to gather opinions from the performers about the backstage facilities, the organisers decide to question a stratified sample of 40 people.

Find how many secondary school children should be selected.
You must show all your working.

[3]

....

....

....

....

Number of secondary school children ....

(b) Of the 128 adult performers, 52 are male and 76 are female.
Gwen decides to interview a stratified sample of **16 adults** and has exactly 16 copies of the questionnaire ready for them.

Using these numbers, she calculates that she should interview 7 male performers and 10 female performers, making a total of **17 adults**.

Explain how this has happened.

[2]

....

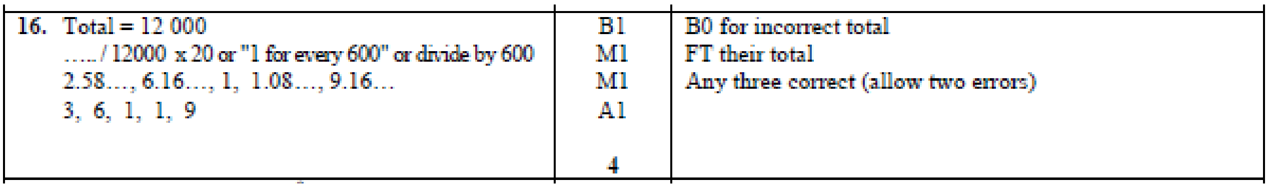
....

....

....

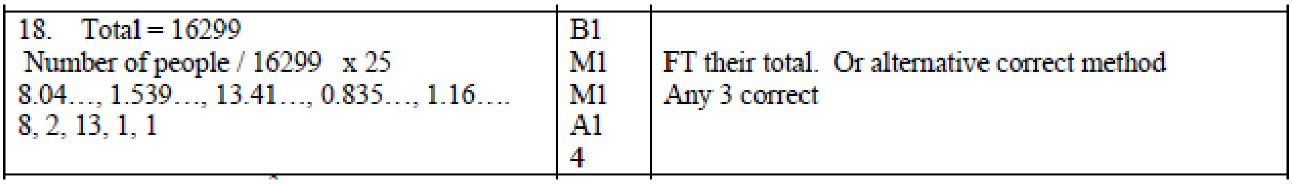
Mark schemes for examination questions on stratified sampling

1. *June 2006 Paper 2 Higher*



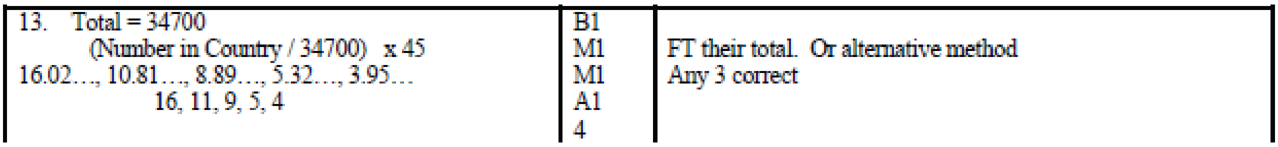
16. Total = 12 000	B1	B0 for incorrect total
….. / 12000 x 20 or "1 for every 600" or divide by 600	M1	FT their total
2.58…, 6.16…, 1, 1.08…, 9.16…	M1	Any three correct (allow two errors)
3, 6, 1, 1, 9	A1	
	4	

2. *June 2007 Paper 2 Higher*



18. Total = 16299	B1	
Number of people / 16299 x 25	M1	FT their total. Or alternative correct method
8.04…, 1.539…, 13.41…, 0.835…, 1.16….	M1	Any 3 correct
8, 2, 13, 1, 1	A1	
	4	

3. *June 2008 Paper 2 Higher*



13. Total = 34700	B1	
(Number in Country / 34700) x 45	M1	FT their total. Or alternative method
16.02…, 10.81…, 8.89…, 5.32…, 3.95…	M1	Any 3 correct
16, 11, 9, 5, 4	A1	
	4	

4. *June 1996 Higher*

Valid factor e.g. consider all year groups, avoid siblings	B1	
Valid reason e.g. need to get a fair representation of all year groups, avoid bias	E1	
	2	

5. *June 2000 Higher*

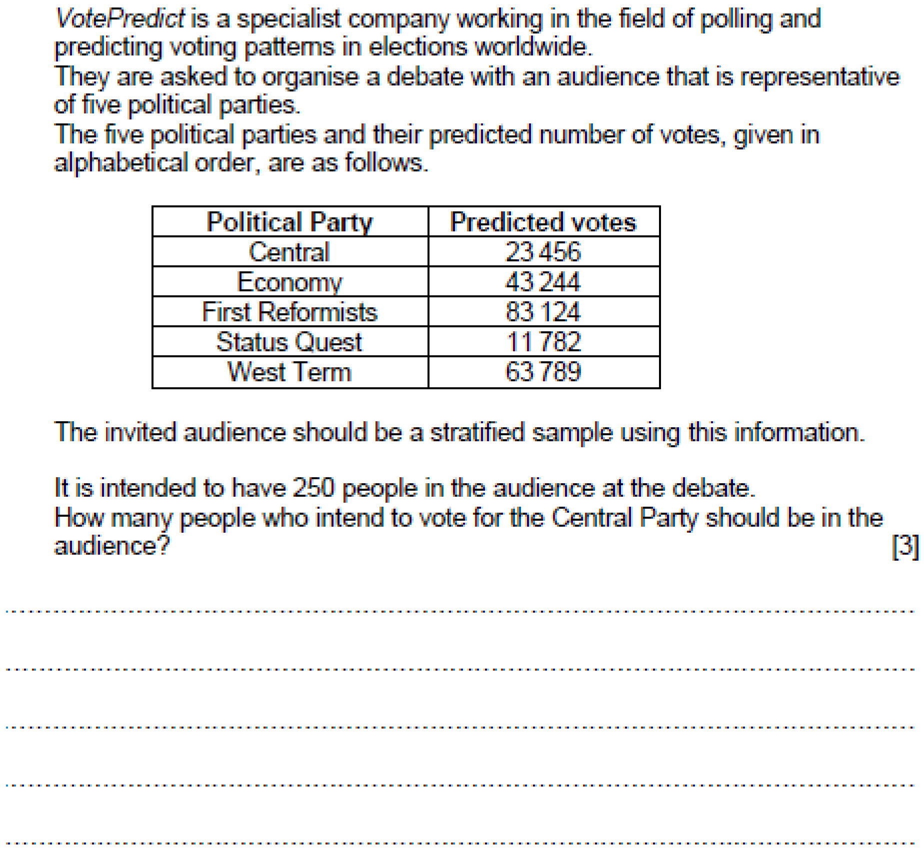
(Numerator =) 190 + 650 (= 840)	M1	
(Denominator =) 190 + 650 + 260 + 900 (= 2000)	M1	
840 / 2000 × 40 000	M1	
= 16 800	A1	
	4	

6. *SAMs 2 Mathematics - Numeracy Unit 1 Higher*

11. (a) (Number of secondary school children =) 73 / (39 + 73 + 128)	M1	Intention to find proportion of 40
73 / 240 × 40 (= 2920 / 240 or 73 / 6 or 12(.1666...) or 12 (1/6))	m1	
= 12	A1	Must be given as a whole number.
(b) 6.5 (male performers) OR 9.5 (female performers)	B1	
Explanation that both numbers have been rounded up.	E1	
	6	

Worked and marked example on stratified sampling

SAMs 1 Mathematics - Numeracy Unit 2 Higher



VotePredict is a specialist company working in the field of polling and predicting voting patterns in elections worldwide.
They are asked to organise a debate with an audience that is representative of five political parties.
The five political parties and their predicted number of votes, given in alphabetical order, are as follows.

Political Party	**Predicted votes**
Central	23 456
Economy	43 244
First Reformists	83 124
Status Quest	11 782
West Term	63 789

The invited audience should be a stratified sample using this information.

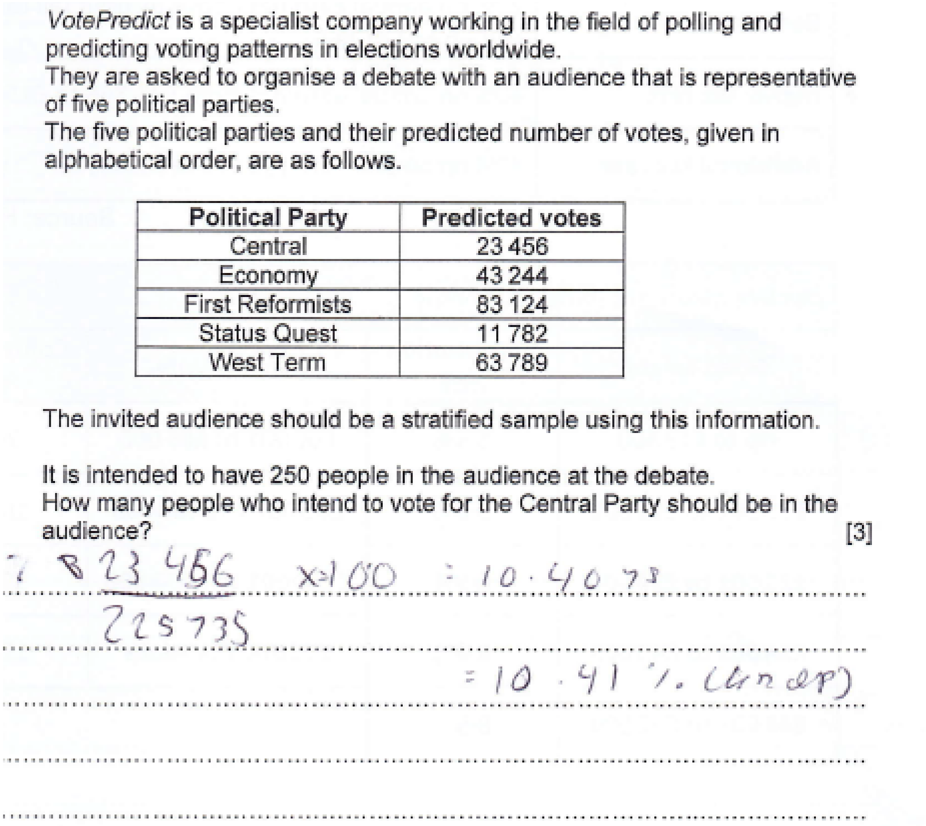
It is intended to have 250 people in the audience at the debate.
How many people who intend to vote for the Central Party should be in the audience? [3]

Mark scheme

(b) $\frac{23456}{23456 + 43244 + 83124 + 11782 + 63789}$	M1	Intention to find Central Party share of the votes
$\frac{23456}{225395} \times 250$	m1	OR sight of 0.104066(194) × 250
26 (people)	A1 4	Must be given as a whole number

Candidate responses

Candidate A.



VotePredict is a specialist company working in the field of polling and predicting voting patterns in elections worldwide.
They are asked to organise a debate with an audience that is representative of five political parties.
The five political parties and their predicted number of votes, given in alphabetical order, are as follows.

Political Party	Predicted votes
Central	23 456
Economy	43 244
First Reformists	83 124
Status Quest	11 782
West Term	63 789

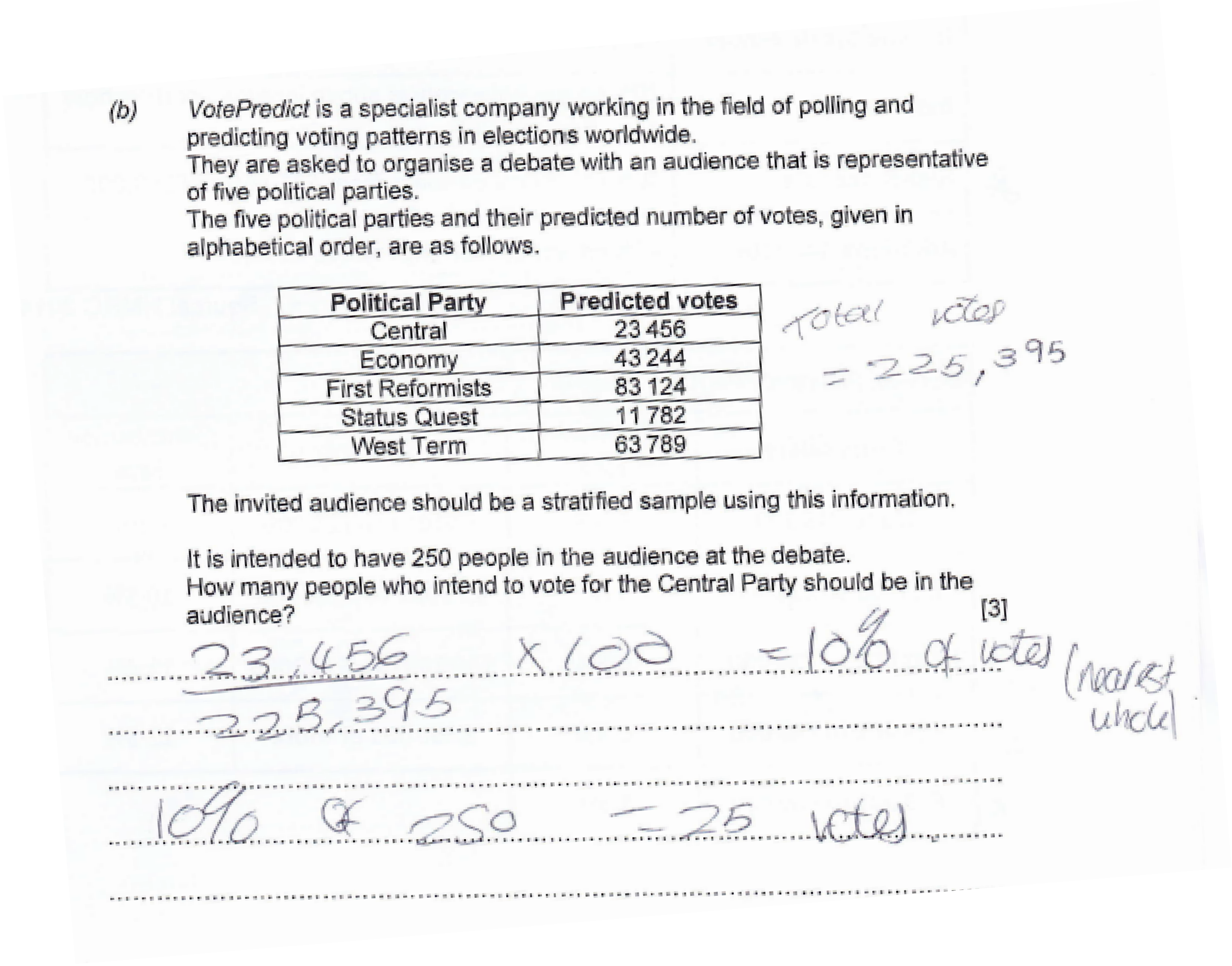
The invited audience should be a stratified sample using this information.

It is intended to have 250 people in the audience at the debate.
How many people who intend to vote for the Central Party should be in the audience? [3]

$\frac{23\,456}{225735} \times 100 = 10.4073$

$= 10.41$ % (4n dp)

Candidate B



(b) *VotePredict* is a specialist company working in the field of polling and predicting voting patterns in elections worldwide.
They are asked to organise a debate with an audience that is representative of five political parties.
The five political parties and their predicted number of votes, given in alphabetical order, are as follows.

Political Party	Predicted votes
Central	23 456
Economy	43 244
First Reformists	83 124
Status Quest	11 782
West Term	63 789

Total votes = 225,395

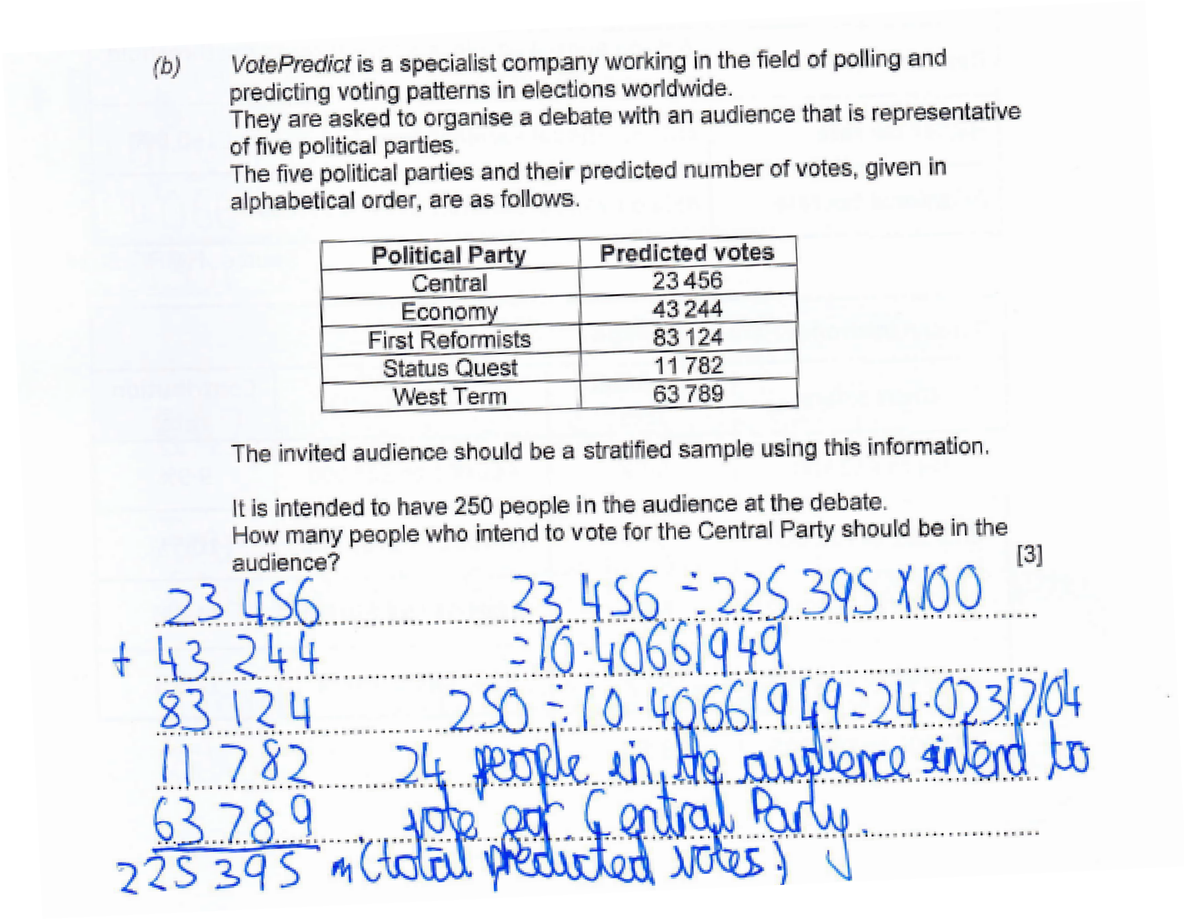
The invited audience should be a stratified sample using this information.

It is intended to have 250 people in the audience at the debate.
How many people who intend to vote for the Central Party should be in the audience? [3]

$\frac{23,456}{225,395} \times 100 = 10\%$ of votes (nearest whole)

10% of 250 = 25 votes

Candidate C



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They are asked to organise a debate with an audience that is representative of five political parties.
The five political parties and their predicted number of votes, given in alphabetical order, are as follows.

Political Party	**Predicted votes**
Central	23 456
Economy	43 244
First Reformists	83 124
Status Quest	11 782
West Term	63 789

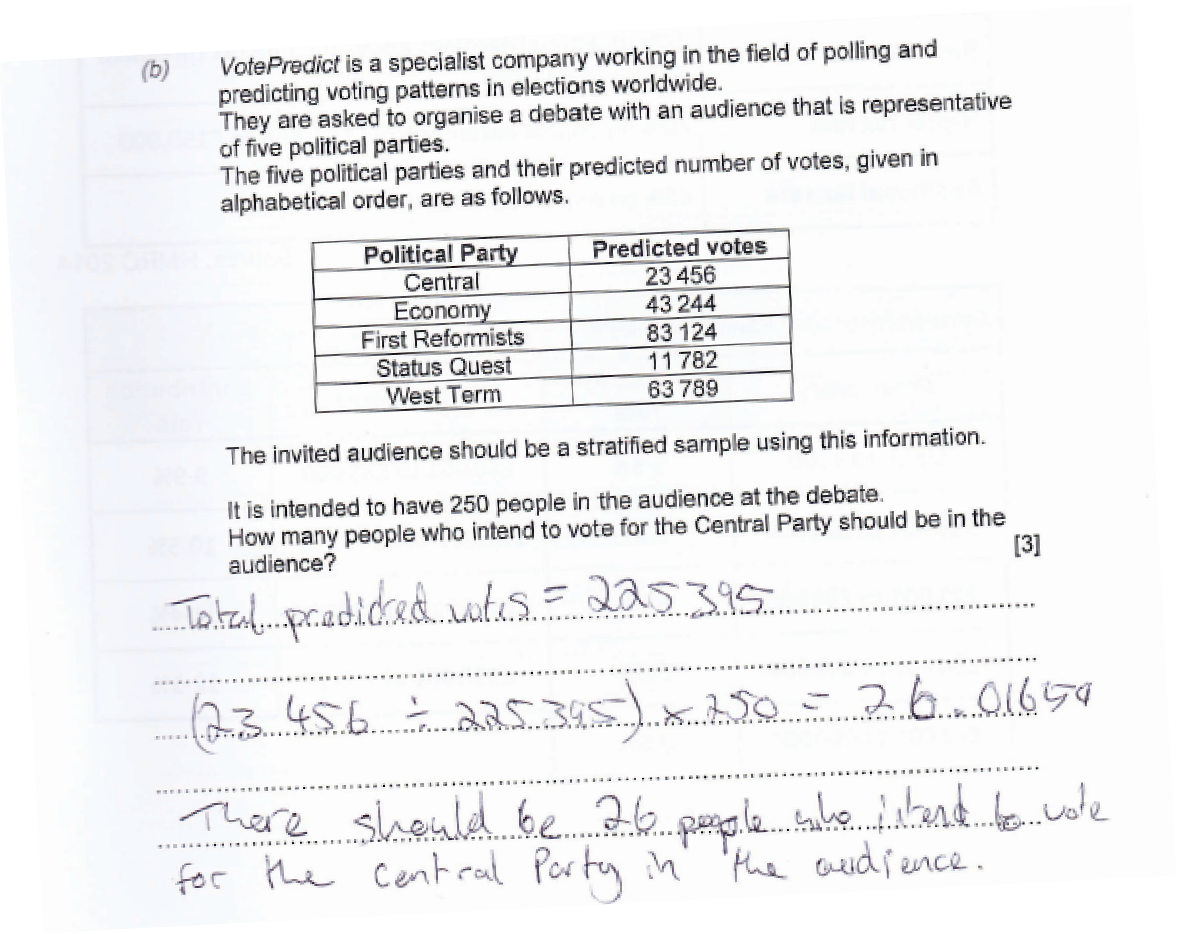
The invited audience should be a stratified sample using this information.

It is intended to have 250 people in the audience at the debate.
How many people who intend to vote for the Central Party should be in the audience? [3]

23 456
+ 43 244
83 124
11 782
63 789
225 395 (total predicted votes)

23 456 ÷ 225 395 x 100
= 10.40661949
250 ÷ 10.40661949 = 24.02317104
24 people in the audience intend to vote for Central Party. ✓

Candidate D



(b) *VotePredict* is a specialist company working in the field of polling and predicting voting patterns in elections worldwide.
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The five political parties and their predicted number of votes, given in alphabetical order, are as follows.

Political Party	Predicted votes
Central	23 456
Economy	43 244
First Reformists	83 124
Status Quest	11 782
West Term	63 789

The invited audience should be a stratified sample using this information.

It is intended to have 250 people in the audience at the debate.
How many people who intend to vote for the Central Party should be in the audience? [3]

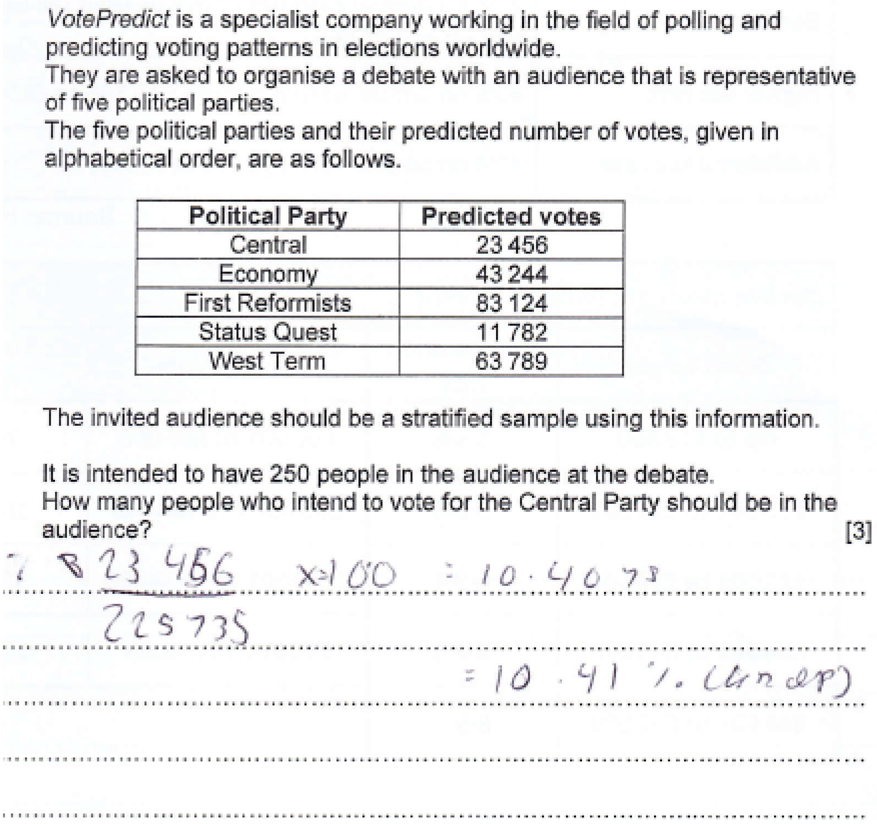
Total predicted votes = 225 395

(23 456 ÷ 225 395) × 250 = 26.01654

There should be 26 people who intend to vote for the central Party in the audience.

Annotated candidate responses

Candidate A.



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First Reformists	83 124
Status Quest	11 782
West Term	63 789

The invited audience should be a stratified sample using this information.

It is intended to have 250 people in the audience at the debate.
How many people who intend to vote for the Central Party should be in the audience? [3]

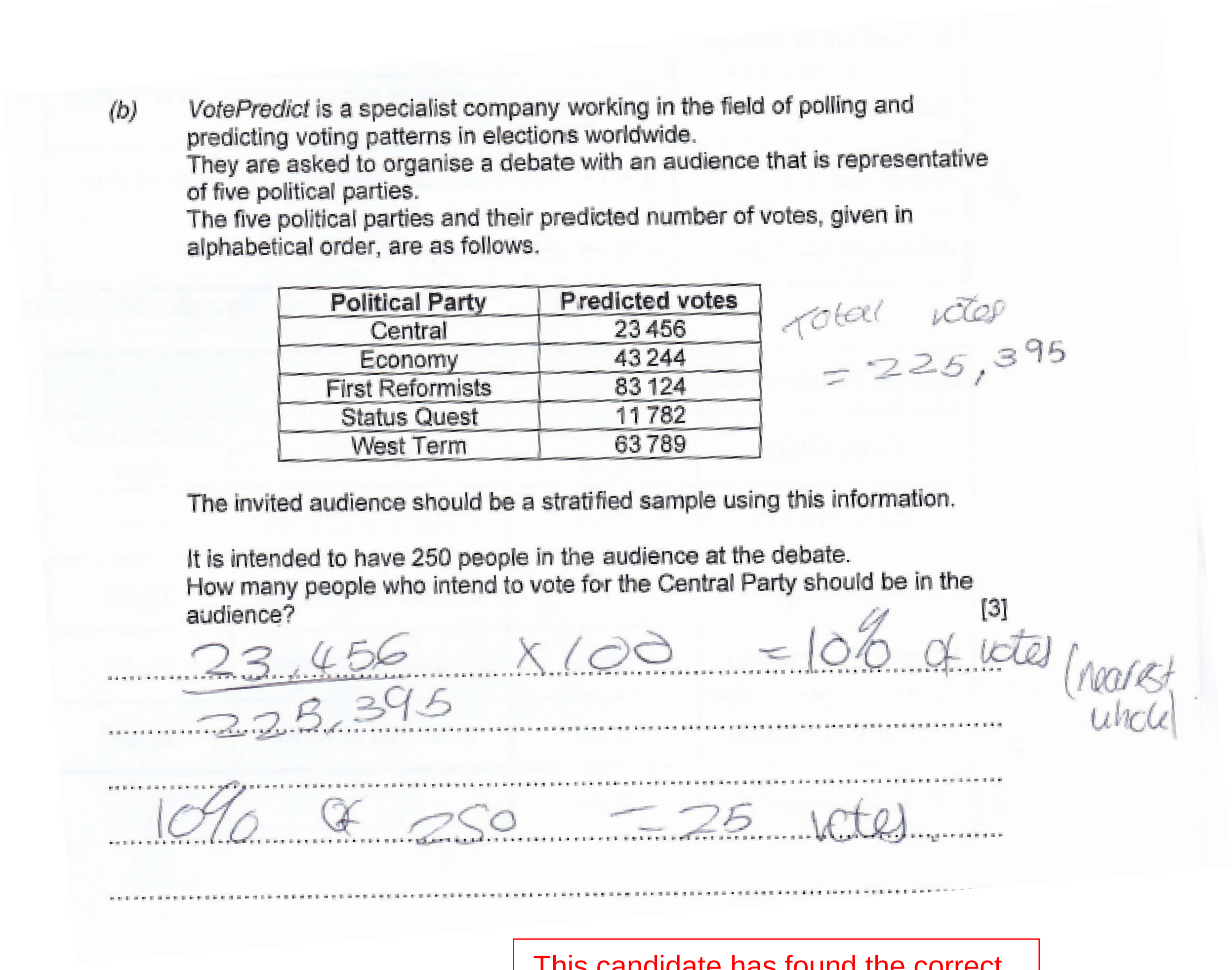
$\% \; \frac{23\,456}{225\,735} \times 100 = 10.4073$

$= 10.41\%$ (2 dp)

This candidate has used a correct method to find the proportion (as a percentage) for the Central party, but has not multiplied this by 250. In fact, a calculator error appears to have resulted in the percentage (given as 10·4073%) being incorrect.

The marks awarded are therefore M1 m0 A0.

Candidate B



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They are asked to organise a debate with an audience that is representative of five political parties.
The five political parties and their predicted number of votes, given in alphabetical order, are as follows.

Political Party	Predicted votes
Central	23 456
Economy	43 244
First Reformists	83 124
Status Quest	11 782
West Term	63 789

Total votes $= 225,395$

The invited audience should be a stratified sample using this information.

It is intended to have 250 people in the audience at the debate.
How many people who intend to vote for the Central Party should be in the audience? [3]

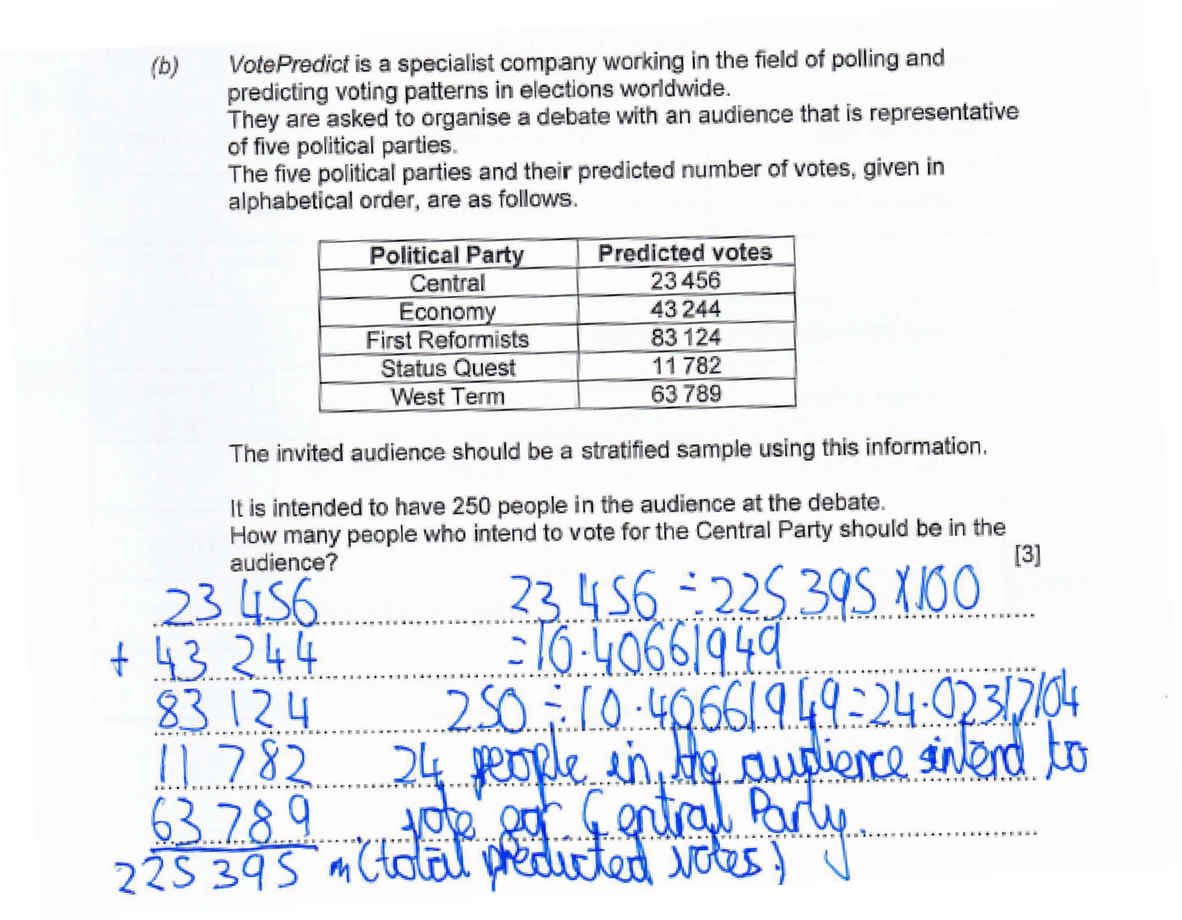
$\frac{23,456}{225,395} \times 100 = 10\%$ of votes (nearest whole)

10% of $250 = 25$ votes

This candidate has found the correct proportion for the Central party, and has multiplied this by 250. However, premature rounding (10·4% to 10%) means a loss of accuracy for the final mark.

The marks awarded are therefore M1 m1 A0.

Candidate C



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The invited audience should be a stratified sample using this information.

It is intended to have 250 people in the audience at the debate.
How many people who intend to vote for the Central Party should be in the audience? [3]

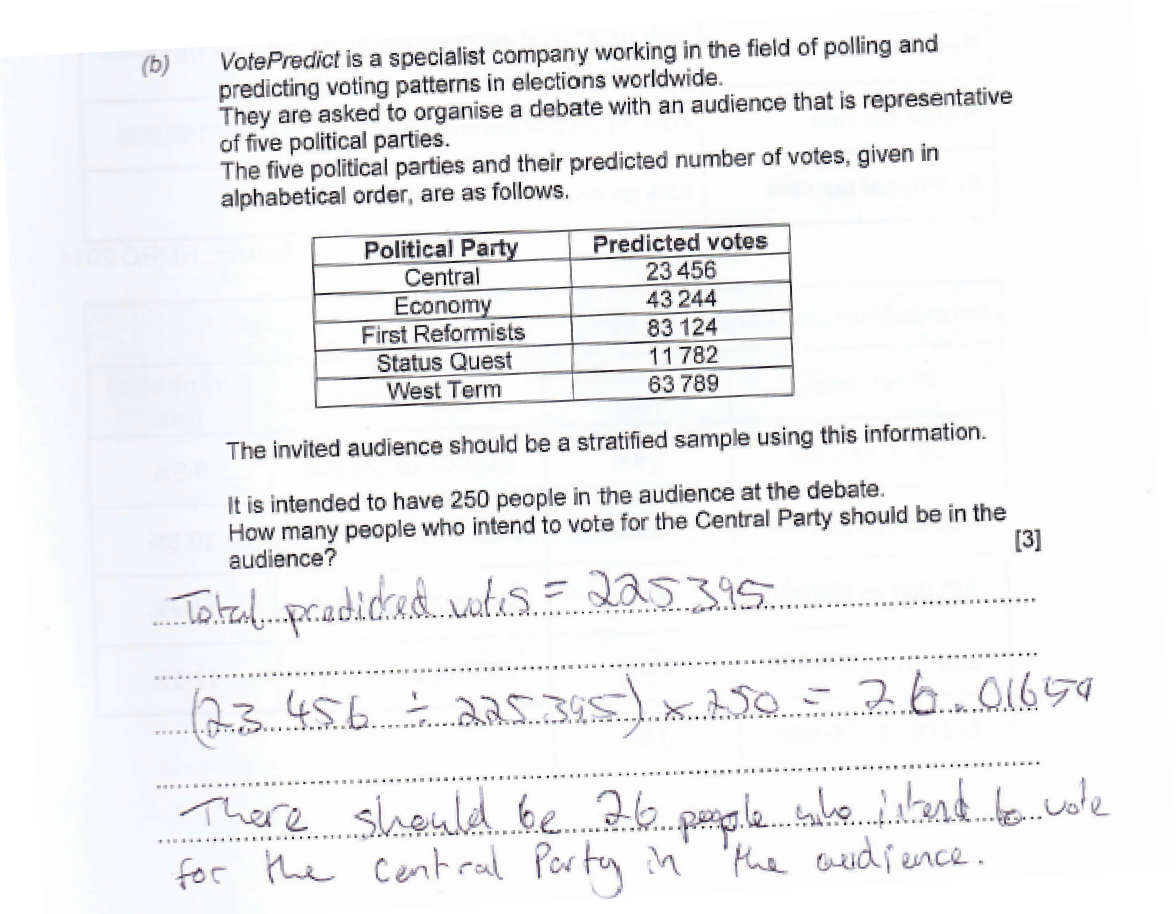
23 456
+ 43 244
83 124
11 782
63 789
225 395 m (total predicted votes)

23 456 ÷ 225 395 x 100
= 10.40661949
250 ÷ 10.40661949 = 24.02317104
24 people in the audience intend to vote for Central Party. ✓

This candidate has found the correct proportion for the Central party, but has then divided this into 250 instead of multiplying.

The marks awarded are therefore M1 m0 A0.

Candidate D



(b) *VotePredict* is a specialist company working in the field of polling and predicting voting patterns in elections worldwide.
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The invited audience should be a stratified sample using this information.

It is intended to have 250 people in the audience at the debate.
How many people who intend to vote for the Central Party should be in the audience? [3]

Total predicted votes = 225395

$(23\,456 \div 225395) \times 250 = 26.01654$

There should be 26 people who intend to vote for the central Party in the audience.

This candidate has found the correct proportion for the Central party, and has correctly multiplied this by 250 to get 26 (rounded to the nearest whole number).

The marks awarded are therefore M1 m1 A1.

Examples of examination questions on random sampling

1. *SAMs 1 Mathematics - Numeracy Unit 2 Higher*

A School Council wants to know pupils' views on their school uniform.
Which of the following statements shows how a truly random sample of the general population can be obtained? [1]
Circle your answer.

A: Randomly selecting pupils in the canteen at lunchtime.

B: Randomly selecting pupils from those that attend the next School Council meeting.

C: Randomly selecting pupils with a surname beginning with the letter J.

D: Giving each pupil a raffle ticket and then randomly drawing raffle tickets for selection.

E: Selecting every 2nd pupil from each form register.

2. *June 1998 Higher*

(a) Use the following extract from a table of random digits to select a random sample of size 4 from a group of 40 people. Start with the first number, and explain your method clearly.

29	83	34	66	00	09	25	51	65	44	88
50	02	13	46	55	97	18	70	95	54	32

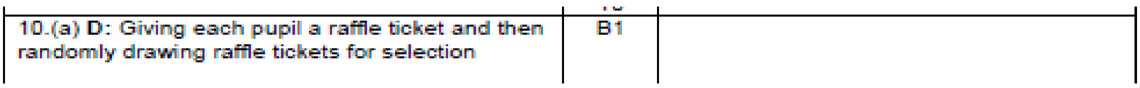
[3]

(b) Tony suggests another method which could be used to select a sample of 4 from the group of 40 people is to make a list of all their names in alphabetical order, and then select every tenth person on the list. Explain briefly why his method will not give a random sample.

[2]

Mark schemes for examination questions on random sampling

1. *SAMs 1 Mathematics - Numeracy Unit 2 Higher*



10.(a) D: Giving each pupil a raffle ticket and then randomly drawing raffle tickets for selection	B1	

2. *June 1998 Higher*

(a) Number the 40 people from 1 to 40. Go through the 2 digit numbers in the random list, writing down any that are between 01 and 40. (Ignore numbers greater than 40 or repeats.) Select the people numbered 29, 34, (0)9, 25	B1 B1 B1 3	Or any 40 different numbers.

4. VOCABULARY OF FINANCE

Specification statement: Mathematics - Numeracy

"Money: The basic principles of personal and household finance, including fuel and other bills, hire purchase, discount, VAT, taxation, best buys, wages and salaries, loan repayments, mortgages, budgeting, exchange rates and commissions.
Simple and compound interest, including the use of efficient calculation methods.
Profit and loss.
Finding the original quantity given the result of a proportional change.
Foreign currencies and exchange rates.
Carrying out calculations relating to enterprise, saving and borrowing, investing, appreciation and depreciation and **understanding annual rates, e.g. AER, APR."**

(Foundation tier content is in standard text;
Intermediate tier content which is in addition to foundation tier content is in underlined text;
Higher tier content which is in addition to intermediate tier content is in **bold** text.)

Topic	**Comment**	**Examples of vocabulary**
Basic ideas of banking	Savings, including maintaining a simple 3 column bank account sheet.	*Bank* account*, credit, debit, balance, withdrawal, deposit, brought forward, carried forward*
Investment	Savings and investments – how they grow.	*Investment, interest rate, compound interest, simple interest, per annum, annual, loan, gross rate, net rate, appreciation, depreciation.*
Personal finance	Basic money management, buying using a credit plan, wages and tax.	*Rent, (utility) bill, credit plan, hire purchase, timesheet, basic rate, overtime rate, fee, callout charge, earnings, tax free, tax rate, income tax, taxable income, threshold, National Insurance, pension contributions, household budget, repayments, discount, cashback, wages, salary, mortgage, personal allowance, gross income.*
Enterprise	Basic money management, depreciation of equipment.	*Depreciation, VAT, expenses, commission, profit, loss.*
Currency transactions	Changing pounds sterling into a foreign currency and vice versa. Comparison of 'true' price by converting to a common currency.	*Exchange, price comparisons, commission*
Inflation	Calculating the effect of inflation on prices and wages.	*Inflation*
*** AER / APR**	***Note that the AER formula is included in the list at the beginning of a question paper.***	***Compound interest, principal, AER, APR, mortgage.***

* See separate notes under 'New Topics'.

5. ADDITIONAL NOTES ON PROPORTION

Topics which are included in both Mathematics - Numeracy and Mathematics but are differently specified

Topic	**Specification statement**	**Mathematics -Numeracy**	**Mathematics only**	**Tier**
Proportion (specified under 'Number')	Direct and inverse proportion.	✓		Intermediate Higher
Proportion (specified under 'Algebra')	Constructing and using equations that describe direct and inverse proportion.		✓	Higher
Venn diagrams (specified under 'Number')	Understanding and using Venn diagrams to solve problems.	✓		Foundation Intermediate Higher
Venn diagrams (specified under 'Statistics')	Use Venn diagrams or other diagrammatic representations of compound events.		✓	Foundation Intermediate Higher

Venn diagrams are dealt with in the section on 'New Content Topics'.

Examples of questions on 'Proportion' follow, according to how they match the specifications.

Examples of questions on proportion for Intermediate and Higher tier GCSE Mathematics - Numeracy or GCSE Mathematics (Number)

1. *November 2012 Unit 1 Higher*

A building firm used 3 machines to concrete an area of 600 m², to a fixed depth, in 5 hours.

The following day they need to concrete a further area of 1120 m², to the same depth, with th work being completed in 4 hours.

Given that all conditions are similar, what is the least number of machines the firm should us on the second day?

………

………

………

……… [3]

2. *January 2013 Unit 1 Higher*

A printer takes 12 hours to complete a job printing 54 000 advertising leaflets using his old print machine.
How long will he take to print another 72 000 similar leaflets using a new machine that works twice as quickly as his old machine?

………

………

………

……… [3]

3. *January 2012 Unit 1 Higher*

It takes 4 hours to empty 6 identical tanks of oil using 15 identical pumps.

How long would it take to empty 2 of these tanks using 3 of these pumps?
Give your answer in hours and minutes.

………

………

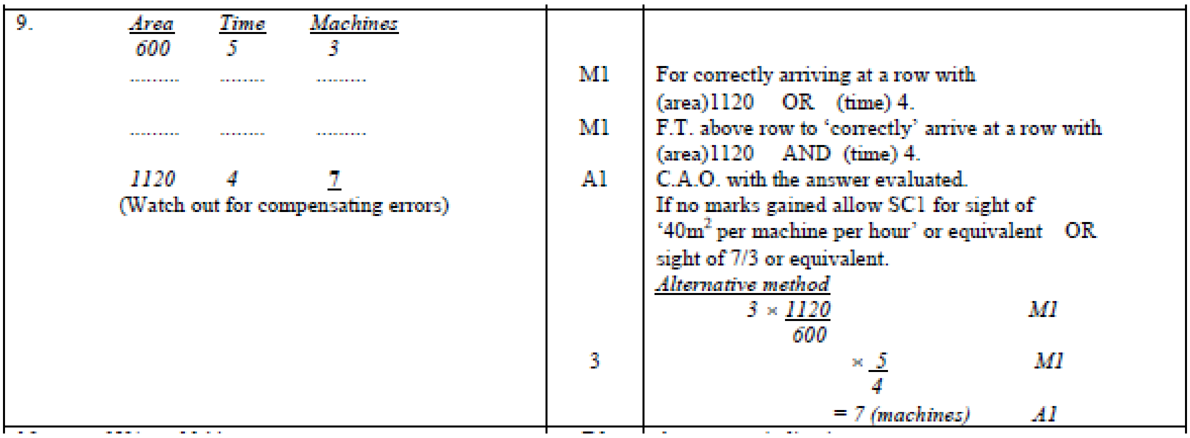
………

………

……… [4]

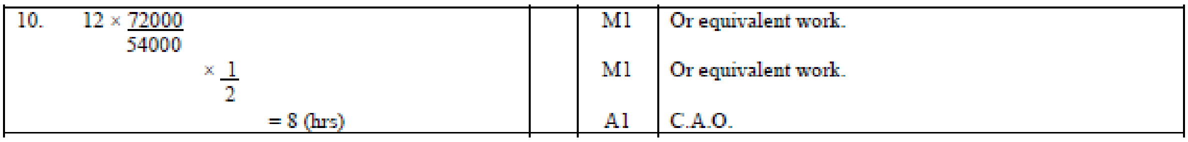
Mark schemes

1. *November 2012 Unit 1 Higher*



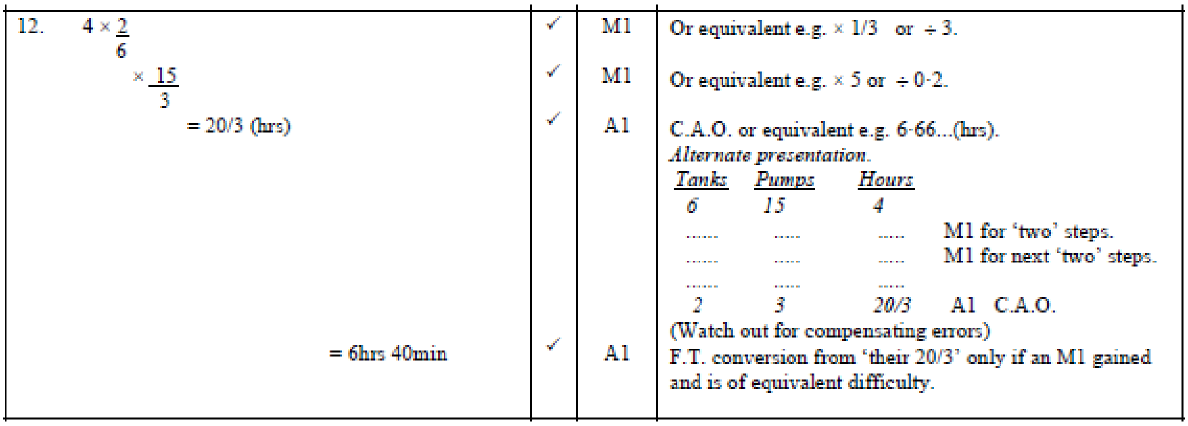
9. *Area* *Time* *Machines* *600* *5* *3* ……… ……… ……… ……… ……… ……… *1120* *4* *7* (Watch out for compensating errors)	M1 M1 A1 3	For correctly arriving at a row with (area)1120 OR (time) 4. F.T. above row to 'correctly' arrive at a row with (area)1120 AND (time) 4. C.A.O. with the answer evaluated. If no marks gained allow SC1 for sight of '40m^2 per machine per hour' or equivalent OR sight of 7/3 or equivalent. *Alternative method* $3 \times \frac{1120}{600}$ *M1* $\times \frac{5}{4}$ *M1* = *7 (machines)* *A1*

2. *January 2013 Unit 1 Higher*



10. $12 \times \frac{72000}{54000}$		M1	Or equivalent work.
$\times \frac{1}{2}$		M1	Or equivalent work.
= 8 (hrs)		A1	C.A.O.

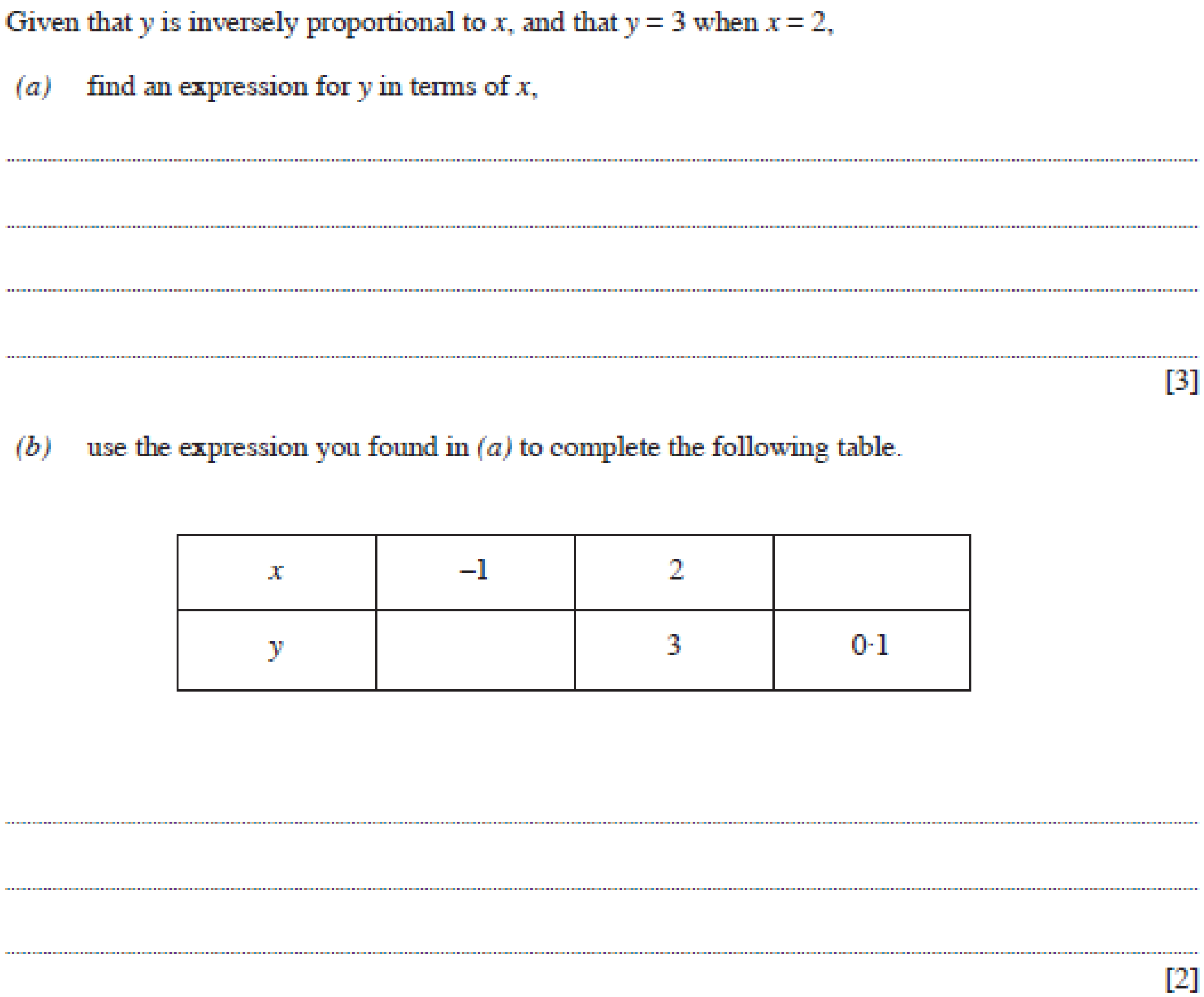
3. *January 2012 Unit 1 Higher*



12. $4 \times \frac{2}{6}$	✓	M1	Or equivalent e.g. × 1/3 or ÷ 3.
$\times \frac{15}{3}$	✓	M1	Or equivalent e.g. × 5 or ÷ 0·2.
= 20/3 (hrs)	✓	A1	C.A.O. or equivalent e.g. 6·66…(hrs). *Alternate presentation.* *Tanks* *Pumps* *Hours* *6* *15* *4* …… …… …… M1 for 'two' steps. …… …… …… M1 for next 'two' steps. …… …… …… *2* *3* *20/3* A1 C.A.O. (Watch out for compensating errors)
= 6hrs 40min	✓	A1	F.T. conversion from 'their 20/3' only if an M1 gained and is of equivalent difficulty.

Examples of questions on proportion for Higher tier GCSE Mathematics only (Algebra)

1. *November 2008 Paper 2 Higher*



Given that y is inversely proportional to x, and that $y = 3$ when $x = 2$,

(a) find an expression for y in terms of x,

[3]

(b) use the expression you found in *(a)* to complete the following table.

x	–1	2	
y		3	0·1

[2]

2. *June 1997 Higher*

A variable y varies directly as the cube of x.

(a) Given that $y = 32$ when $x = 2$, find the formula connecting y and x.

....

....

....

....
[3]

(b) Find the value of x when $y = 4000$.

....

....
[1]

3. *June 2007 Paper 1 Higher*

Given that y is inversely proportional to x^2, and that $y = 4$ when $x = 10$,

(a) find an expression for y in terms of x,

....

....

....

....
[3]

(b) calculate

(i) the value of y when $x = 20$,

....

....

....
[1]

(ii) a value of x when $y = \frac{1}{100}$.

....

....

....

....
[2]

4. *June 1997 Higher*

A plank rests horizontally on two supports, one at each end. When an object of mass m kg is placed on the centre of the plank, the centre sinks a distance d cm. It is known that d is proportional to $\sqrt{m}$.

(a) Using k as a constant of proportionality, write down an equation for d in terms of m.

....

[1]

(b) In an experiment to determine the value of k, the following results were obtained.

Mass, m kg	4	8	12	14
Distance, d cm	0·45	0·60	0·78	0·87

Substitute **each** of the four pairs of values given in the table into the equation you have found in *(a)* and hence estimate the value of k.

....

....

....

....

....

....

....

....

....

[3]

(c) Using your estimate for k, write down the formula connecting d and m.

....

[1]

(d) Use your formula to estimate the distance that the centre of the plank will sink if an object of mass 10 kg is placed on its centre.

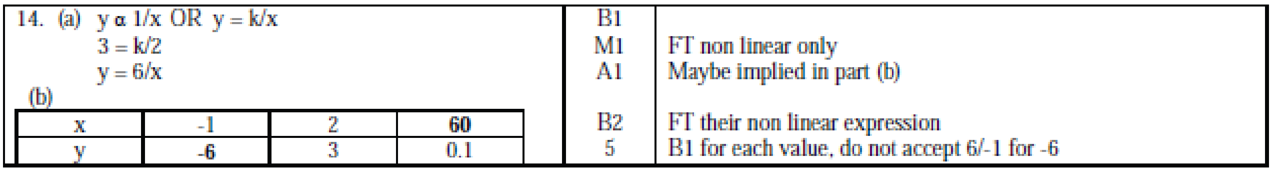
....

....

[1]

Mark schemes

1. *November 2008 Paper 2 Higher*



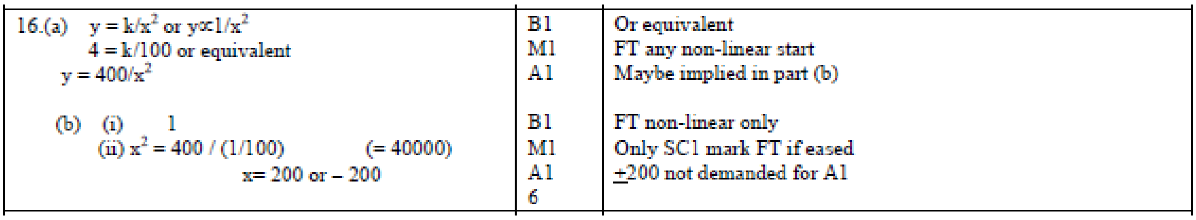
14. (a) $y \alpha 1/x$ OR $y = k/x$	B1	
$3 = k/2$	M1	FT non linear only
$y = 6/x$	A1	Maybe implied in part (b)
(b)	B2 5	FT their non linear expression B1 for each value, do not accept 6/-1 for -6

x	-1	2	**60**
y	**-6**	3	0.1

2. *June 1997 Higher*

(a) $y = kx^3$ or $y \alpha x^3$	B1	
$32 = k \times 2^3$	M1	
$y = 4x^3$	A1	
(b) 10	B1 4	

3. *June 2007 Paper 1 Higher*



16.(a) $y = k/x^2$ or $y \propto 1/x^2$	B1	Or equivalent
$4 = k/100$ or equivalent	M1	FT any non-linear start
$y = 400/x^2$	A1	Maybe implied in part (b)
(b) (i) 1	B1	FT non-linear only
(ii) $x^2 = 400 / (1/100)$ (= 40000)	M1	Only SC1 mark FT if eased
$x = 200$ or $- 200$	A1 6	$\pm$200 not demanded for A1

4. *June 1997 Higher*

(c) $d = k\sqrt{m}$	B1	
(d) $k = 0.45 / \sqrt{4} = 0.225$ and $k = 0.60 / \sqrt{8} = 0.212$ and $k = 0.78 / \sqrt{12} = 0.225$ and $k = 0.87 / \sqrt{14} = 0.233$	B2	All 4 values found. B1 for 2 or 3 values found.
$k = 0.2$ or 0.22	B1	An estimate based on rounding which agrees to 1 decimal place, or based on the mean of the 4 values.
(e) $d = 0.2\sqrt{m}$ or $d = 0.22\sqrt{m}$	B1 5	

6. ORGANISING, COMMUNICATING AND WRITING ACCURATELY

Explanation

In each examination paper, candidates will be assessed on their ability to organise, communicate and write accurately (OCW).
The assessment of OCW will be separate from the assessment of the mathematics, and whilst it is impossible to completely detach the assessment of OCW from the assessment of the mathematics in a question, the OCW marks will not depend on whether or not marks have been awarded for the mathematics. However, the assessment of OCW has to be on mathematics that is relevant to the question. Therefore it is likely, but not impossible, that if no marks are awarded for the mathematics, then OCW could not be assessed in that response and no marks would be awarded for OCW.

OCW is split into two strands:

1) **Organising and Communicating** (OC)

In order to gain the OC mark, candidates will need to organise their response to a question in a coherent and logical manner. They will need to communicate their response well. Their response will need to be relevant to the question asked. This means that candidates will need to:

- present their response in a structured way,
- explain to the reader what they are doing at each step of their response. In some questions, a label or brief description may be enough. In others, a full explanation may be more appropriate,
- lay out their explanations and working in a way that is clear and logical, so that the reader can easily follow their response,
- write a conclusion that draws together their results and explains to the reader what their answer means.

Note: candidates who don't explain their steps but instead write a long paragraph after their working will NOT gain the OC mark. Similarly, one long continuous paragraph that includes working will not gain the OC mark as this is not an appropriate logical, coherent way to communicate mathematics.

2) **Writing Accurately** (W)

In order to gain the W mark, candidates will need to communicate their response accurately. Their response will need to be relevant to the question asked.
This means they will need to:

- make few, if any, errors in spelling, punctuation and grammar,
- show all their working,
- use correct mathematical form in their working,
- use appropriate terminology, units, etc.

Note: if a candidate does not give labels, explanations, working etc., then it will be difficult to assess how accurately they have communicated their response. In a question where it is desirable for responses to include explanations, candidates should include them. If there is insufficient opportunity for us to assess writing accurately in a response, then we will not be able to award the mark. However, it will be possible, in some cases, to award the W mark when the OC mark has not been awarded.

It may be that in question papers, the OC mark is awarded in one question and the W mark is awarded in another. The above points apply to these questions too. In particular, for the W mark, candidates will need to show sufficient working and/or explanations to enable examiners to assess the accuracy of their writing.

How OCW is shown on the examination papers

On each question paper, it will be clear which question(s) will be assessing OCW.

On papers where the OC mark and the W mark are assessed in the same question:

- The following statement will be in the 'Information For Candidates' section on the first page:
  *The assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing in question **1.***

- The following statement will be at the beginning of the question or part-question:
  You will be assessed on the quality of your organisation, communication and accuracy in writing in this question.

- The marks for the question will be shown like this: [5 + OCW 2]
  This means that 5 marks will be allocated to the mathematics and 2 marks to the assessment of OCW.

On papers where the OC mark and the W mark are assessed in different questions:

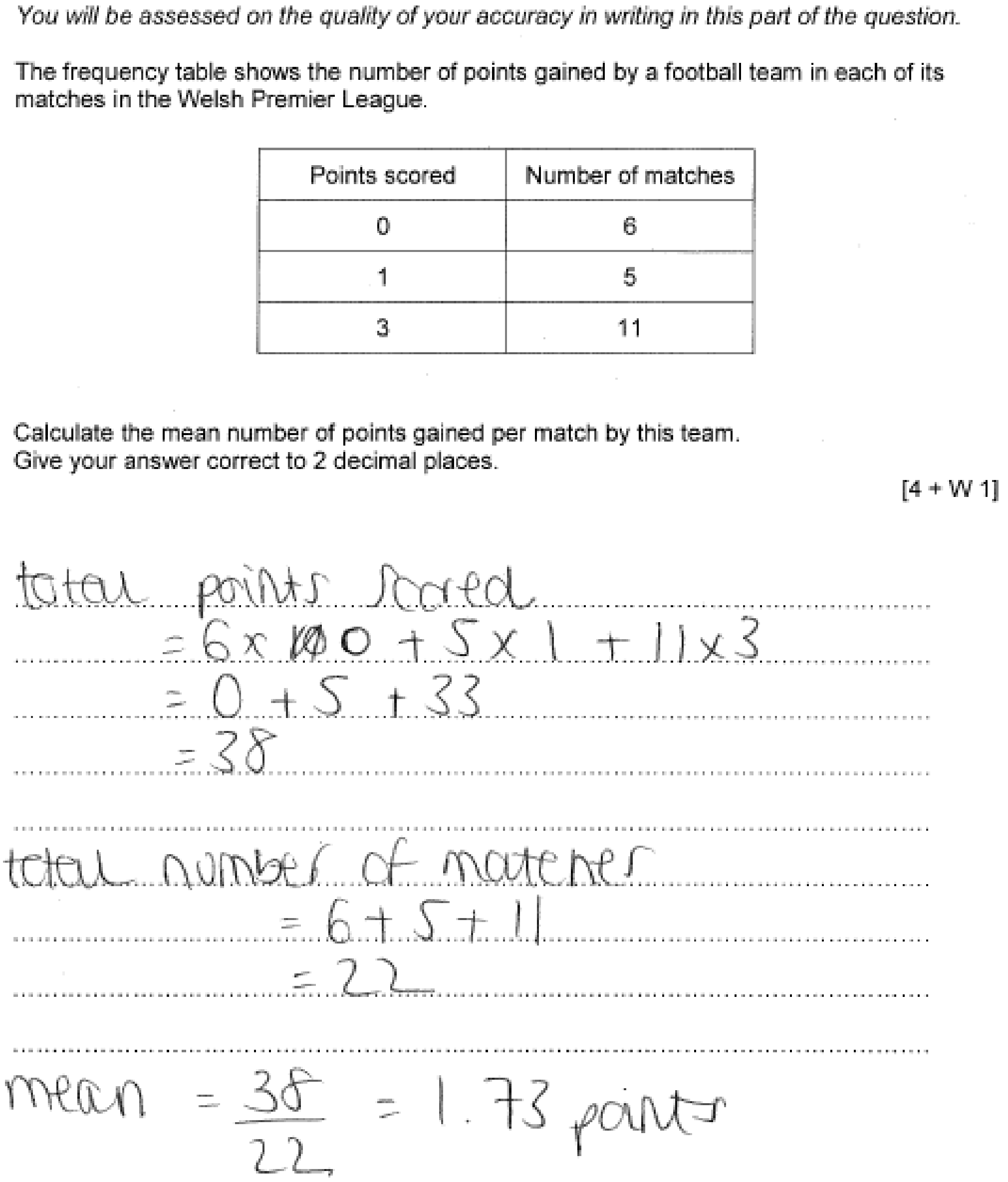
- The following statements will be in the 'Information For Candidates' section on the first page:
  *The assessment will take into account the quality of your linguistic and mathematical organisation and communication in question **5(c).***
  *The assessment will take into account the accuracy of your writing (linguistic and mathematical) in question **14.***
- The following statement will be at the beginning of the question or part-question in which <u>organising and communicating</u> will be assessed:
  You will be assessed on the quality of your organisation and communication and in this question.
- The following statement will be at the beginning of the question or part-question in which <u>writing accurately</u> will be assessed:
  You will be assessed on the quality of your accuracy in writing in this question.
- The marks for the questions will be shown like this: [4 + OC 1] and [7 + W 1], for example.

In the second set of sample assessment materials, on GCSE Mathematics Unit 2 Foundation Tier, organising and communicating is assessed in one question (Question 5(c)) and accuracy in writing in another (Question 14).

Examples of Good Practice

For individual examples in examination papers, discussion will be had as to what constitutes losing the OC or W marks, and these will be discussed with examiners in marking conferences. Therefore, it isn't appropriate to comment here on what candidates would have to do to **lose** these marks. Here are examples of responses that clearly deserve OC1 and/or W1.

1. *SAMs 2 Mathematics Unit 2 Foundation*



You will be assessed on the quality of your accuracy in writing in this part of the question.

The frequency table shows the number of points gained by a football team in each of its matches in the Welsh Premier League.

Points scored	Number of matches
0	6
1	5
3	11

Calculate the mean number of points gained per match by this team.
Give your answer correct to 2 decimal places.

[4 + W 1]

total points scored

$= 6 \times \cancel{10}\, 0 + 5 \times 1 + 11 \times 3$

$= 0 + 5 + 33$

$= 38$

total number of matches

$= 6 + 5 + 11$

$= 22$

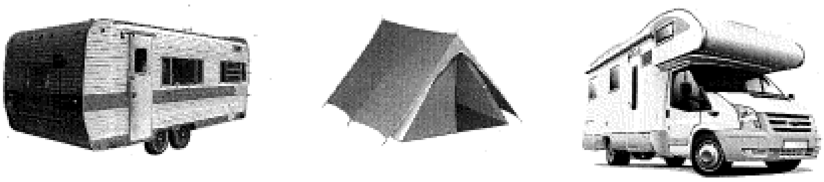
mean $= \frac{38}{22} = 1.73$ points

In this question, the candidate scores 4 marks for a fully correct response.
In addition, a mark for accuracy in writing can be awarded.
Here, the candidate is awarded W1 as the response is written using correct English, but (and perhaps more importantly in this particular question) the candidate uses correct mathematical from in their response.
No mark is given in this question for organising and communicating.

2. *SAMs 2 Mathematics – Numeracy Unit 1 Foundation*

You will be assessed on the quality of your organisation, communication and accuracy in writing in this question.

Marine Bay
West Wales
Camping & Caravan Park



Pitch fees per night.
Tent = £12
Caravan = £16
Motor-home = £15

The Jones family invited their friends, the Williams and the Phillips families to stay at the Marine Bay Camping and Caravan Park, West Wales.

The Jones family have a caravan and stayed for 3 nights.
The Williams family have a motor-home and only stayed for one night.
The Phillips family stayed in a tent.

The total fee for the 3 pitches was £99.

For how many nights did the Phillips family stay?
You must show all your working.

[4 + OCW 2]

Jones family pays 3×16 = £48
Williams family pays £15

Phillips family pays 99−48−15 = £36

Number of nights = 36 ÷ 12 = 3

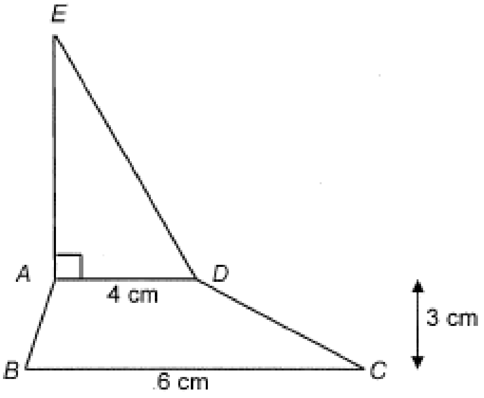
$$\begin{array}{r} 16 \\ \times\ 3 \\ \hline 48 \\ \hline \end{array}\qquad \begin{array}{r} 99 \\ -\ 48 \\ \hline 51 \\ \hline \end{array}\qquad \begin{array}{r} {}^{4}\cancel{5}\,{}^{1}1 \\ -\ 1\ 5 \\ \hline 36 \\ \hline \end{array}$$

This candidate is awarded 4 marks for the mathematics as the method and answer are both correct.
The candidate is awarded OC1 as they have organised their work in a coherent manner, shown their working and labelled each line of their response.
They are awarded W1 as the response is written using correct English, correct mathematical form is used in their working, and units are used appropriately.

3. *SAMs 2 Mathematics Unit 1 Intermediate*

You will be assessed on the quality of your organisation, communication and accuracy in writing in this question.

A right-angled triangle *ADE* is attached to a trapezium *ABCD* as shown below.



This candidate is awarded 5 marks for the mathematics as the methods and answers are correct.

Diagram not drawn to scale

AD = 4 cm, *BC* = 6 cm, and the perpendicular height of the trapezium is 3 cm.
The triangle and the trapezium have equal area.

Calculate the length of *AE*.

[5 + OCW 2]

Area of trapezium

$$= \left(\frac{4+6}{2}\right) \times 3$$

$$= 5 \times 3$$

$$= 15\text{cm}^2$$

So area of △ is equal to the area of the trapezium

therefore area of △ = 15cm²

$$\frac{AE \times 4}{2} = 15$$

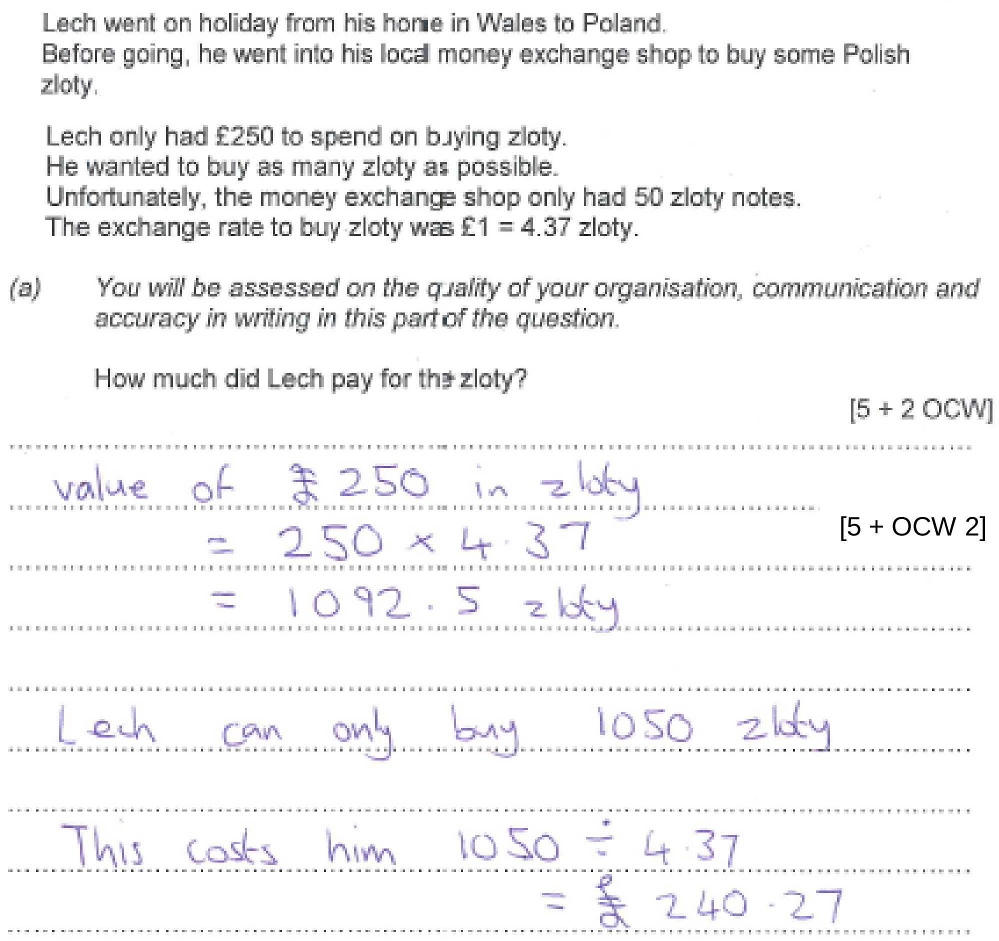
$$AE \times 2 = 15$$

$$AE = \frac{15}{2}$$

$$AE = 7.5\text{cm}$$

The candidate is awarded OC1 as their response is structured and coherent, with steps labelled. Even though there is no sentence at the end explaining their answer, what is given is self-explanatory.
They are awarded W1 as the response is written using correct English, correct mathematical form is used in their working, and units are used appropriately.

4. *SAMs 2 Mathematics – Numeracy Unit 2 Higher*



Lech went on holiday from his home in Wales to Poland.
Before going, he went into his local money exchange shop to buy some Polish zloty.

Lech only had £250 to spend on buying zloty.
He wanted to buy as many zloty as possible.
Unfortunately, the money exchange shop only had 50 zloty notes.
The exchange rate to buy zloty was £1 = 4.37 zloty.

(a) *You will be assessed on the quality of your organisation, communication and accuracy in writing in this part of the question.*

How much did Lech pay for the zloty?

[5 + 2 OCW]

value of £ 250 in zloty

= 250 × 4·37

= 1092·5 zloty

Lech can only buy 1050 zloty

This costs him 1050 ÷ 4·37

= £ 240·27

[5 + OCW 2]

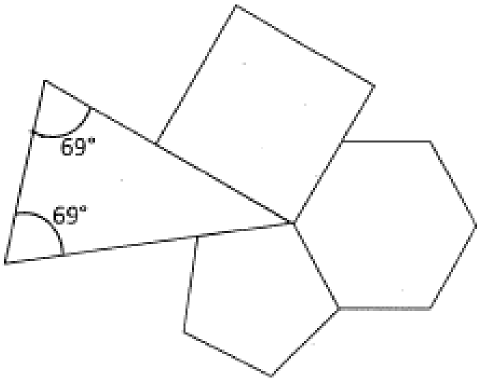
This candidate is awarded 5 marks for the mathematics.
The candidate is awarded OC1 as they have organised their work in a coherent manner, shown their working and labelled each line of their response.
They are awarded W1 as the response is written using correct English, correct mathematical form is used in their working, and units are used appropriately.

5. *SAMs 2 Mathematics Unit 1 Higher*

You will be assessed on the quality of your organisation, communication and accuracy in writing in this question.

Prove that it is possible for a square, a regular pentagon, a regular hexagon and an isosceles triangle with two equal angles of 69° to meet at a point as shown below.

[6 + OCW 2]



This candidate is awarded 6 marks for the mathematics. The candidate is awarded OC1 as they have organised their work in a coherent manner, shown their working and labelled each line of their response. They are awarded W1 as the response is written using correct English, correct mathematical form is used in their working, and units are used appropriately.

Angle total in a hexagon

= 4 x 180

= 720°

Angle total in a pentagon

= 3 x 180

= 540°

The four angles at the point are

△ 180° − 69° − 69° = 42°

□ 360° ÷ 4 = 90°

⬠ 540° ÷ 5 = 108°

⬡ 720° ÷ 6 = 120°

108
5|540

120
6|7'20

Total for the four angles

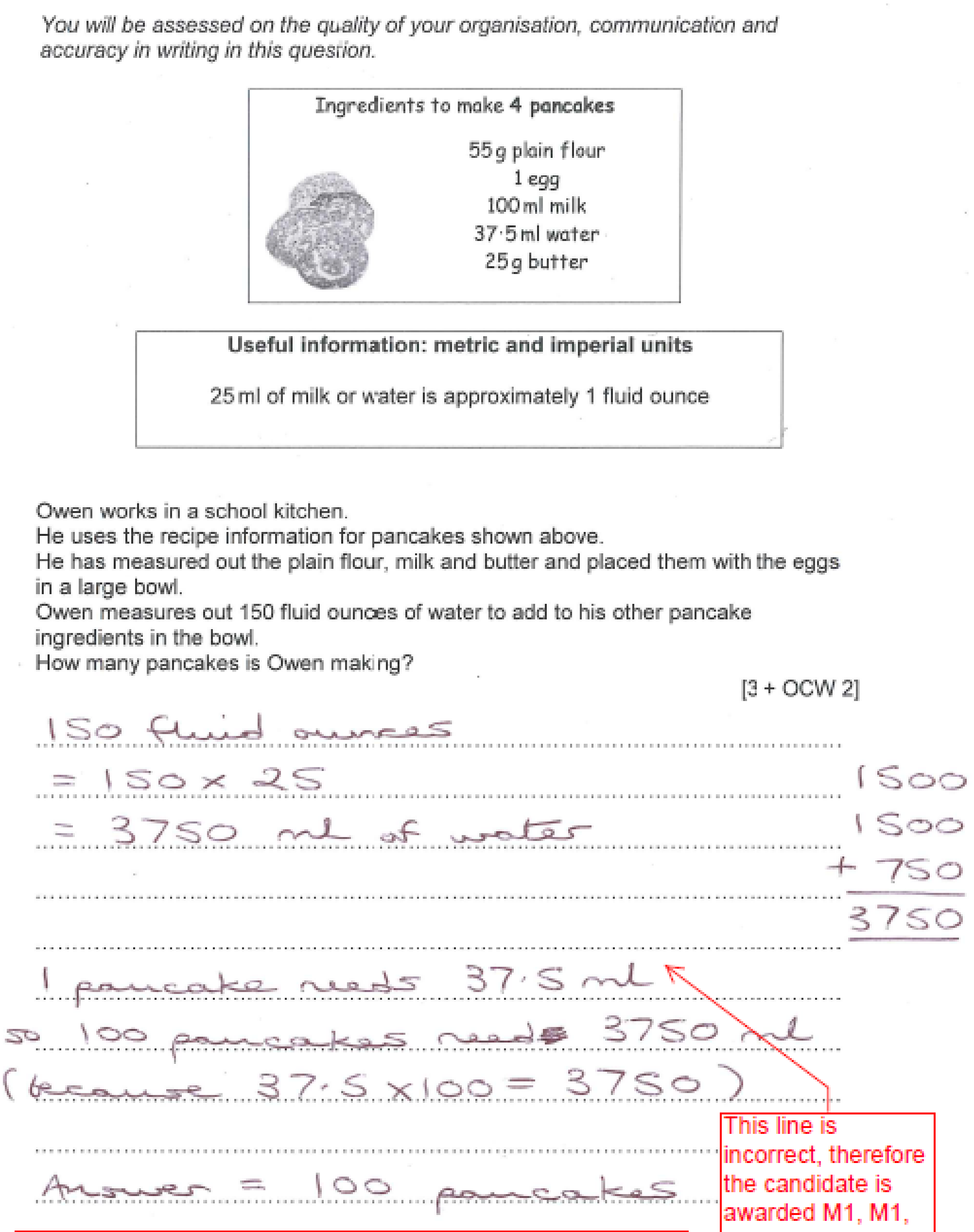
= 42 + 90 + 108 + 120

= 360°

42
90
108
120
360
11

So it is possible.

6. *SAMs 2 Mathematics – Numeracy Unit 1 Higher*



You will be assessed on the quality of your organisation, communication and accuracy in writing in this question.

Ingredients to make 4 pancakes

55 g plain flour
1 egg
100 ml milk
37·5 ml water
25 g butter

Useful information: metric and imperial units

25 ml of milk or water is approximately 1 fluid ounce

Owen works in a school kitchen.
He uses the recipe information for pancakes shown above.
He has measured out the plain flour, milk and butter and placed them with the eggs in a large bowl.
Owen measures out 150 fluid ounces of water to add to his other pancake ingredients in the bowl.
How many pancakes is Owen making?

[3 + OCW 2]

150 fluid ounces
= 150 × 25
= 3750 ml of water

1500
1500
+ 750
3750

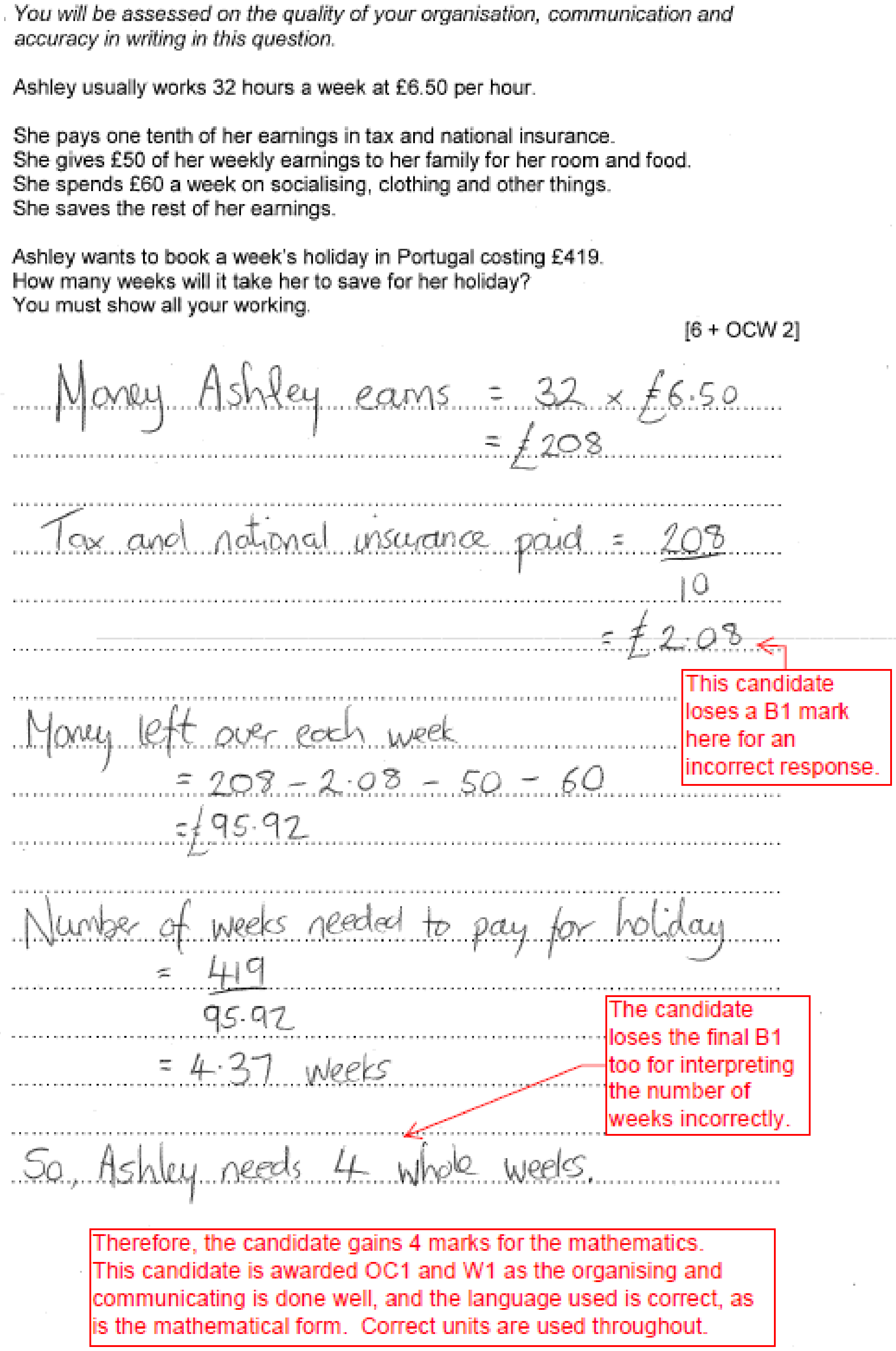
1 pancake needs 37·5 ml
so 100 pancakes needs 3750 ml
(because 37·5 x 100 = 3750)

Answer = 100 pancakes

This line is incorrect, therefore the candidate is awarded M1, M1, A0.

Even though the final answer is incorrect, this candidate is awarded OC1 and W1 as the organising and communicating is done well, and the language used is correct, as is the mathematical form. Correct units are used throughout.

7. *SAMs 2 Mathematics – Numeracy Unit 2 Foundation*



You will be assessed on the quality of your organisation, communication and accuracy in writing in this question.

Ashley usually works 32 hours a week at £6.50 per hour.

She pays one tenth of her earnings in tax and national insurance.
She gives £50 of her weekly earnings to her family for her room and food.
She spends £60 a week on socialising, clothing and other things.
She saves the rest of her earnings.

Ashley wants to book a week's holiday in Portugal costing £419.
How many weeks will it take her to save for her holiday?
You must show all your working.

[6 + OCW 2]

Money Ashley earns = 32 × £6.50
= £208

Tax and national insurance paid = $\frac{208}{10}$
= £2.08

This candidate loses a B1 mark here for an incorrect response.

Money left over each week
= 208 − 2.08 − 50 − 60
= £95.92

Number of weeks needed to pay for holiday
= $\frac{419}{95.92}$
= 4.37 weeks

The candidate loses the final B1 too for interpreting the number of weeks incorrectly.

So, Ashley needs 4 whole weeks.

Therefore, the candidate gains 4 marks for the mathematics.
This candidate is awarded OC1 and W1 as the organising and communicating is done well, and the language used is correct, as is the mathematical form. Correct units are used throughout.

7. NEW QUESTION STYLES

Multiple choice questions

Answering multiple choice questions will usually involve choosing between five options for 1 mark only (even if there is sometimes a need for more than one step to reach the answer).

The incorrect answers, or distractors, will usually include those which arise from common errors or misconceptions.

Showing working is not required, though there may be some space provided for this in some cases - appropriate use of the writing space should be encouraged in order to avoid the temptation to 'guess'.

Candidates should understand that 'circle the correct answer' means they should not select more than one option.

Examples

1. *SAMs 2 Mathematics Unit 1 Foundation and Intermediate*

Circle the correct answer for each of the following statements.

(a) 0·2 is equivalent to

2% 20% 0·2% $\frac{1}{5}\%$ $\frac{2}{10}\%$

[1]

(b) 5·4 – 2·16 is equal to

2·24 3·24 3·34 3·36 7·56

[1]

(c) $\frac{5}{6} - \frac{1}{3}$ is equal

$\frac{51}{63}$ $\frac{4}{3}$ $\frac{1}{2}$ $\frac{4}{6}$ 0·43

[1]

2. *SAMs 2 Mathematics Unit 2 Foundation and Intermediate*

(a) Circle the correct answer for each of the following statements.

(i) Helen has bought one of the eighty tickets sold in a raffle. The probability that Helen wins the top prize in the raffle is

$\frac{1}{79}$ 1% 1:80 $\frac{1}{80}$ 80%

[1]

(i) One ball is selected at random form a box containing 5 blue balls, 4 red balls and 1 yellow ball. The probability that the selected ball is blue is

$\frac{5}{5}$ $\frac{1}{2}$ $\frac{5}{41}$ $\frac{10}{5}$ 5%

[1]

(b) A bag contains some red, green and black beads.
One bead is selected at random from the bag.

The probability of selecting a green bead from the bag is $\frac{1}{3}$.

Which of the following sets of beads could have been in the bag?
Circle the correct answer.

2 red	3 red	3 red	7 red	5 red
1 green	6 green	3 green	4 green	3 green
1 black	3 black	4 black	1 black	4 black

[1]

3. *SAMs 2 Mathematics Unit 1 Intermediate and Higher*

Circle the correct answer for each of the following statements.

(a) The gradient of the line $2y = 4x + 3$ is

$\frac{1}{2}$ $\qquad$ $\frac{3}{2}$ $\qquad$ $\frac{2}{3}$ $\qquad$ $\frac{3}{4}$ $\qquad$ 2

[1]

(b) The line $3y = 5x - 6$ crosses the y-axis *at*

$y = -2$ $\qquad$ $y = -\frac{1}{2}$ $\qquad$ $y = 2$ $\qquad$ $y = \frac{5}{3}$ $\qquad$ $y = \frac{1}{2}$

[1]

(c) The point with coordinates

$(3 , -2)$ $\qquad$ $(0 , 2)$ $\qquad$ $(-3 , 2)$ $\qquad$ $(2 , 3)$ $\qquad$ $(3 , 7)$

lies on the line $y = 3x - 2$.

[1]

4. *SAMs 2 Mathematics Unit 1 Higher*

(a) Which one of the following numbers is rational? Circle your answer.

[1]

π $\quad\quad$ $\sqrt{2}$ $\quad\quad$ $\sqrt[3]{16}$ $\quad\quad$ $\sqrt[3]{\frac{125}{8}}$ $\quad\quad$ $\sqrt[4]{20}$

(b) Which one of the following numbers is irrational? Circle your answer.

[1]

$\left(\frac{3}{8}\right)^2$ $\quad\quad$ $\sqrt{144}$ $\quad\quad$ $\sqrt[3]{64}$ $\quad\quad$ $0\cdot\dot{7}912\dot{5}$ $\quad\quad$ π^2

5. *SAMs 2 Mathematics Unit 2 Higher*

(a) Circle your answer in each of the following.

(i) $\sqrt{200}$ simplifies to

20 $\quad\quad$ $10\sqrt{2}$ $\quad\quad$ $20\sqrt{10}$ $\quad\quad$ $100\sqrt{2}$ $\quad\quad$ $2\sqrt{10}$

[1]

(ii) $\sqrt{5} + \sqrt{45}$ simplifies to

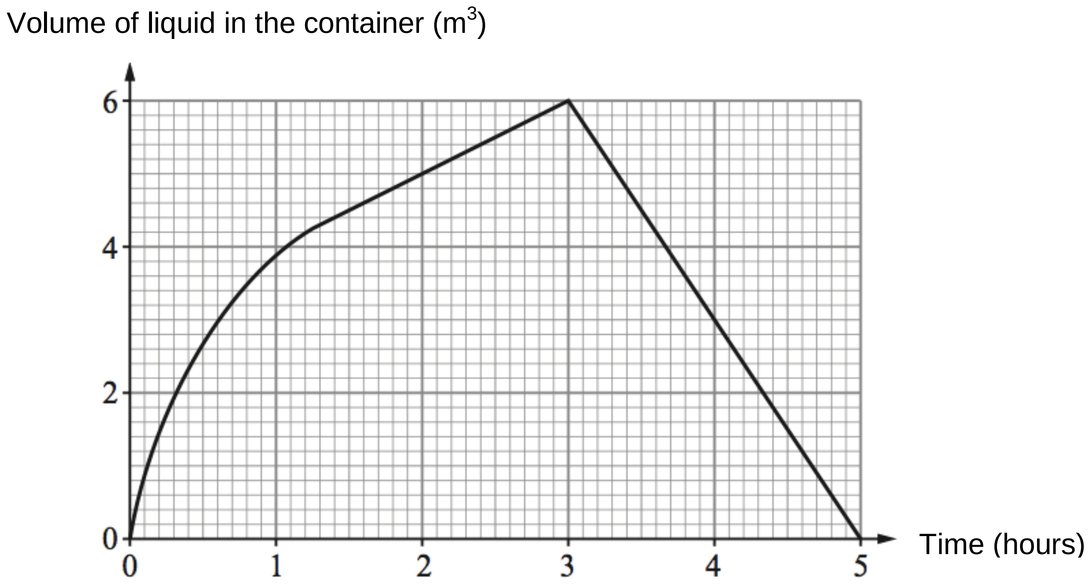
$\sqrt{50}$ $\quad\quad$ $\sqrt{225}$ $\quad\quad$ $4\sqrt{5}$ $\quad\quad$ $10\sqrt{5}$ $\quad\quad$ $4\sqrt{10}$

[1]

Multiple choice questions may involve using reasoning to choose a correct statement.

6. *SAMs 2 Mathematics - Numeracy Unit 2 Foundation and Intermediate*

The graph shows the process of a container being filled with liquid and emptied into a tanker.



Put a tick in the box next to the correct statement.

[1]

The container fills at a constant rate from when it is empty to when it is full.	
The container fills at a constant rate to start with, then slows down.	
After starting to fill, the rate at which the container fills up increases.	
The container starts to fill quickly, then slows down to a constant rate.	
It is not possible to tell whether or not the rate at which the tank fills up remains the same.	

True / False questions

A question will involve approximately five statements, each of which needs to be classified as TRUE or FALSE.

There will generally be 2 marks awarded for all parts correct, with 1 mark for all but one part correct.

7. *SAMs 2 Mathematics Unit 2 Foundation*

(a) Circle either TRUE or FALSE for each statement given below. [2]

STATEMENT		
A cuboid has 6 vertices.	TRUE	FALSE
A tetrahedron is a pyramid with 4 triangular faces only.	TRUE	FALSE
A cube has 12 equal edges.	TRUE	FALSE
A triangular prism has 3 rectangular faces.	TRUE	FALSE

8. *SAMs 2 Mathematics Unit 2 Intermediate and Higher*

Circle either TRUE or FALSE for each statement given below.

[2]

STATEMENT		
Circles with diameters of equal length are congruent.	TRUE	FALSE
Regular pentagons whose perimeters are of equal length are congruent.	TRUE	FALSE
Scalene triangles that have the same three angles are congruent.	TRUE	FALSE
Rectangles with equal areas are congruent.	TRUE	FALSE

9. *SAMs 1 Mathematics - Numeracy Unit 2 Foundation and Intermediate*

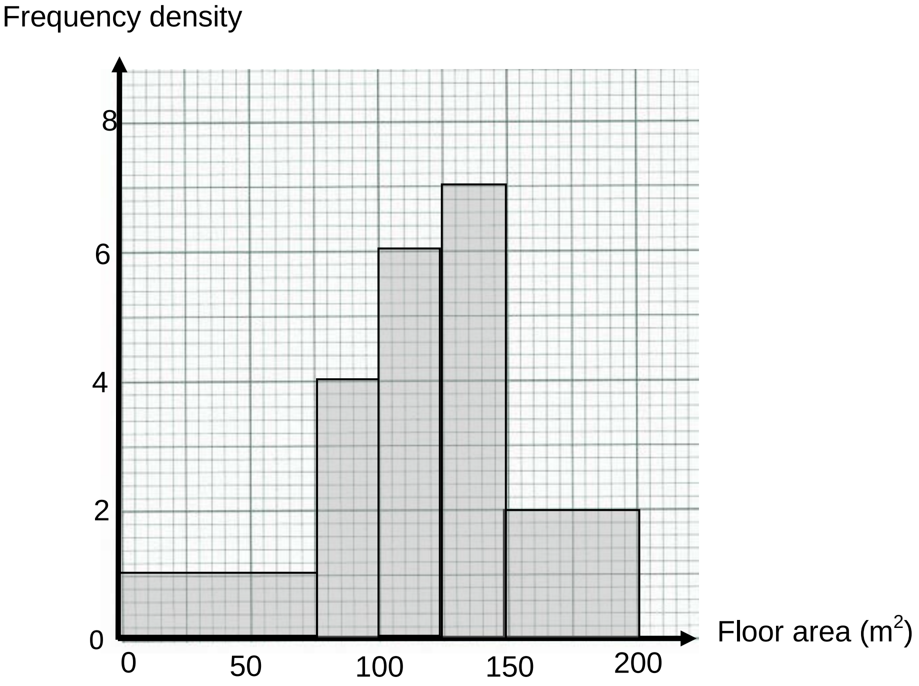
. There were 32 rugby players in the 2013 – 2014 Wales rugby squad.
The mean height of these rugby players was 189 cm.

Circle either TRUE or FALSE for each of the following statements. [2]

All the rugby players in the squad must have been taller than 189 cm.	TRUE	FALSE
If there was a rugby player of height 191 cm in the squad, there must have been a rugby player of height 187 cm.	TRUE	FALSE
The majority of the rugby players in the squad must have been of height 189 cm.	TRUE	FALSE
If some of the rugby players in the squad were taller than 189 cm, then some must have been shorter than 189 cm.	TRUE	FALSE
Half the rugby players in the squad must have been shorter than 189 cm, and half of the rugby players in the squad must have been taller than 189 cm.	TRUE	FALSE

10. *SAMs 2 Mathematics – Numeracy Unit 1 Higher*

The histogram illustrates the floor areas of the offices available to let by *Office Space Wales* letting agency.



Circle either TRUE or FALSE for each of the following statements.

[2]

There are definitely no offices available with less than 10 m^2 of space.	TRUE	FALSE
The modal class of office space is between 125 m^2 and 150 m^2.	TRUE	FALSE
The number of offices over 100 m^2 is double the number under 100 m^2.	TRUE	FALSE
There is enough information in the histogram to allow us to calculate an exact value for the mean office space.	TRUE	FALSE
The number of offices under 50 m^2 is definitely the same as the number over 175 m^2.	TRUE	FALSE

Questions which involve interpreting extended information

11. *SAMs 2 Mathematics – Numeracy Unit 2 Foundation and Intermediate*

Boat owners are charged to keep their boats in a harbour.

Charges for a North Wales harbour are given in the table below.

Period	Price per metre (£ per metre) **exclusive** of VAT	Notes
Annual	320	Minimum length of boat 9 m
Six monthly	180	Minimum length of boat 7 m
Monthly	40	No minimum length

Notes

- VAT is charged at a rate of 20%.
- All charges are **per metre**; any part metre is charged as a complete metre.
- Combinations of the periods are allowed.
  For example, for exactly 7 months, pay for 6 months then pay for an extra month, or pay monthly for each of the 7 months.

(a) **Including VAT**, how much would the **monthly** charge be for a 10 m boat? Circle your answer.

[1]

£40 £48 £400 £480 £4800

(b) **Excluding VAT**, how much would the **six monthly** charge be for an 8·2 m boat?

[1]

12. *SAMs 1 Mathematics - Numeracy Unit 1 Foundation and Intermediate*

Dragon CarCare is a car cleaning company.

Dragon CarCare is charged the following costs for products and services.

Car cleaning products	**Costs**
Car wash liquid	£1 per 5 litre bottle
Window spray	£2 per 2 litre bottle
Wax	£2.50 per 2 litre drum
Cloths and sponges	10 p each

Service	**Unit cost**
Water	£2 per m^3 + Standing charge £4 per month
Electricity	25p per kWh + Standing charge £10 per month + 5% VAT

During June *Dragon CarCare* used the following quantities of products.

Car cleaning products	**Quantity used**
Car wash liquid	12 bottles
Window spray	8 bottles
Wax	6 drums
Cloths and sponges	100 cloths + 100 sponges

At the beginning and at the end of June, the meter readings for water and electricity were recorded.

Service	**Time: 00:01** **Date: 1 June 2014** **Meter reading**	**Time: Midnight** **Date: 30 June 2014** **Meter reading**
Water	3450 m^3	3950 m^3
Electricity	3000 kWh	3800 kWh

(a) How much did *Dragon CarCare* spend on car cleaning products in June 2014? [3]

(b) Calculate the total cost of the water and electricity used by *Dragon CarCare* during June 2014. [4]

(c) The operating costs for *Dragon CarCare* is the sum of the water costs, the electricity costs and the cost of the products used.

Calculate the operating costs for *Dragon CarCare* for June 2014 [1]